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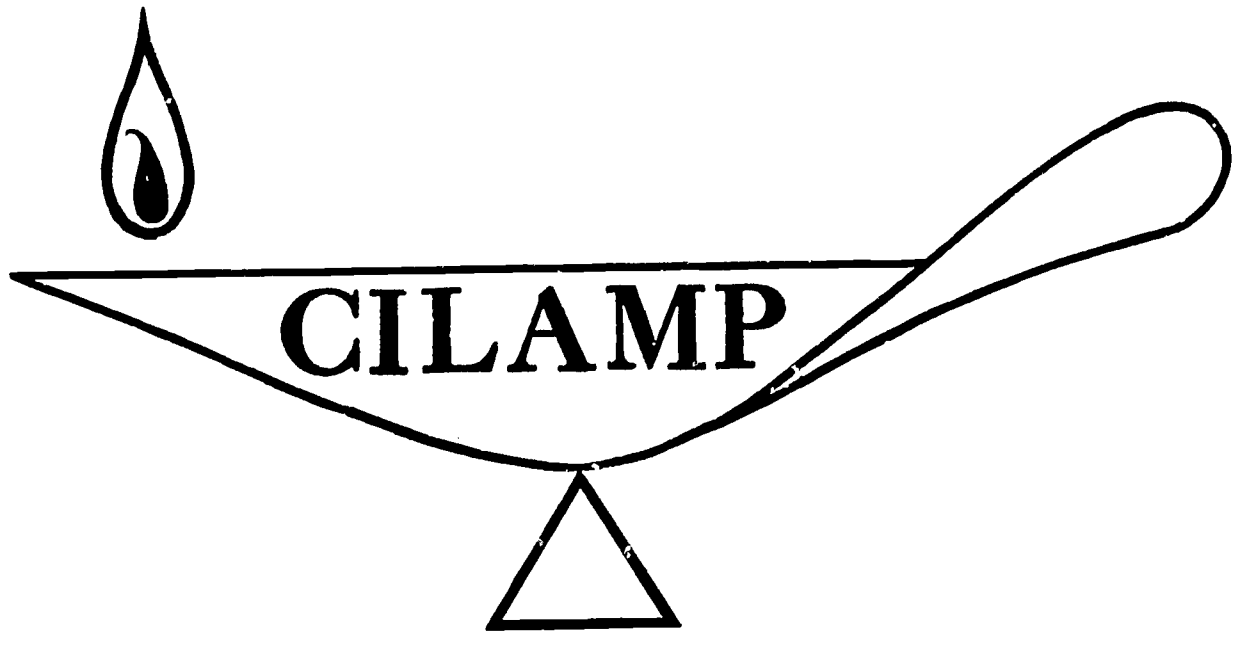
The materials in this Enrichment Student Project (ESP) are designed especially for the low achiever student in mathematics. The booklet is a self contained unit consisting of four elements--a mathematical puzzle, a set of instructions, response sheets, and a suitable container for keeping the unit together. ESP is a motivational idea aimed at attracting the student's interest and promoting his involvement in a portion of mathematics that can be enjoyed. The materials which have been collected for this ESP, complete with solution of problems for the teacher's convenience involve peg, dissection, cube, and topology puzzles. This work was prepared under ESEA Title III contract. (RP)



Central Iowa Low Achiever Mathematics Project

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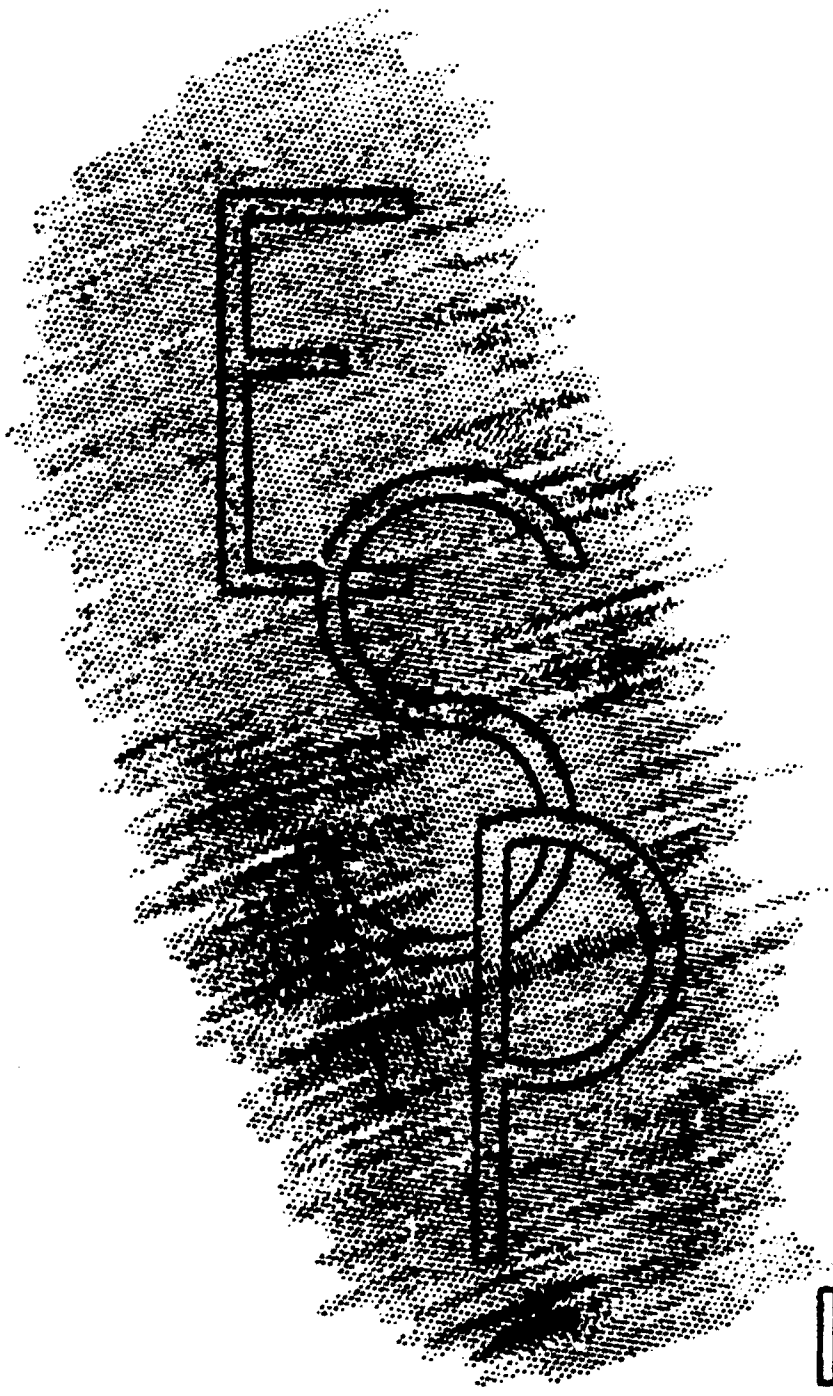

BY JOSEPH ZIMMERMAN

CENTRAL IOWA LOW-ACHIEVER MATHEMATICS PROJECT
(CILAMP)
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TABLE OF CONTENTS

ESP NOTES

PUZZLE	Commercial	Homemade	Individual	Class	Level of Difficulty: Fair	Hard	Very Hard
*Ten Men In A Boat	x	x	x	x	x		
Reversal	x	x	x				x
Quadrix		x	x		x		
*Elimination Plus	x	x	x				x
Elimination Square	x	x	x				x
Elimination Triangle	x	x	x				x
*Garage Shuffle		x	x		x		
*Tanagrams	x	x	x	x	x		
*Dissected Square		x	x	x	x		
*Dissected "T"		x	x		x		
Cross Dissections		x	x			x	
Star Trick		x	x		x		
*Butterfly Puzzle		x	x	x		x	
Hexed	x	x	x				x
Voodoo	x	x	x	x	x		
*Soma Cube	x	x	x		x		
*Combo	x	x	x				x
*Gambler's Die		x	x		x		
Paper Cube		x	x	x	x		
Oriental Cube	x		x		x		
*Wire Heart	x	x	x		x		
*Shoestring Puzzle		x	x		x		
Kite Knot		x	x		x		
The Gordian Knot		x	x			x	
Another Knot		x	x		x		
Ring Release		x	x		x		
*Moebius Strips		x	x	x	x		
*Tower of Hanoi		x	x	x	x		
*Puzzling Pyramid	x	x	x	x	x		
Double Cross		x	x	x		x	
Three X Three	x	x	x	x	x		
Five X Five		x	x	x	x		
*Stick Puzzles		x	x	x	x		
Loco	x	x	x		x		
Digit	x		x		x		
Mystery Maze	x		x		x		

*Indicates those puzzles which are highly recommended for a minimal classroom set of ESP's.

I: INTRODUCTION

E S P

What is it?

An Enrichment Student Project is a self-contained unit composed of four elements: a mathematical puzzle, a set of instructions, response sheets, and a suitable container. The puzzle can be almost anything that is a problem solving situation. There are hundreds of possibilities, some commercially made and others homemade. A few of the possibilities are given in the pages which follow, but these are intended only as an introduction or beginning. The user should feel free to add new ones as well as to alter the old ones. The instructions, typed on index cards, should be as clear and as brief as possible. From the instructions a student should be able to learn the object of the puzzle, the rules that govern or restrict a solution, and any pertinent questions that should be answered in the process.

The response sheet is just a blank half-sheet of paper which the student uses, if he so desires, to record his solution or to answer the questions. And, finally, the puzzle, instruction cards, and response sheets are enclosed in a suitable container. A cigar box, shoe box, manilla envelope, or other small box makes it possible to keep the unit together and adds an element of mystery to the puzzle.

Why use it?

As the name would imply, one of the reasons for using Enrichment Student Projects is to enrich the student's study of mathematics. But more than that E S P is a motivational idea aimed at catching the student's interest and involving him in a portion of mathematics that can be enjoyed. One of the biggest problems any mathematics teacher faces is the negative attitude many students have towards mathematics in general. This attitude must be dealt with before any learning can take place. Once the student's interest has been aroused by

the puzzle, then the related mathematics can be drawn out. Also, E S P serves as an incentive for students to work, since it can be used as a reward when the normal work has been completed. And, finally, E S P is a way of providing for the individual differences of the students. There are many puzzles where a slow student is on an equal footing with a good student when it comes to finding a solution, and as a result is in as good a position to find success. In summary, then, E S P is a way of enriching, motivating, involving, changing attitudes, rewarding, and providing for success.

How can it be used?

There are as many different ways of using E S P as there are teachers who have used it. The following are but a few of the ways which teachers have felt were successful.

First of all, E S P can be used on an individual basis, when a student finishes an assignment before the majority of the class and asks, "What can I do now?" Keeping the set of E S P's in a convenient corner will make it possible for a student to get the teacher's permission and to find a puzzle with as little confusion as possible. Requiring a student to get permission first discourages a student from rushing through and doing his work in a slipshod manner.

Some teachers keep a chart with a class list on one side and the list of E S P's across the top. Each student is assigned one of the puzzles and when he has solved it a check mark is recorded. The chart serves as an incentive to master a given puzzle and in so doing to receive recognition. The same kind of recognition can be given by putting up signs on a bulletin board naming the first one to solve a given puzzle or the champion on one of the puzzles, for example: Mary Kay -- 1st to solve the Tower of Hanoi with 8 discs, or John Brown -- 20 Men in a Boat.

Secondly, E S P can be used with an entire class. Sometimes the whole class needs a pick-me-up or a shot-in-the-arm before anything can be accomplished.

If you have a classroom set of Enrichment Student Projects you can have them passed out and allow fifteen or twenty minutes for each student to become familiar with a different puzzle. It is recreational and the students enjoy it, but it is also mathematical and the students learn from it.

Some of the E S P's are such that the same puzzle can be used by the whole class at the same time. Most of these involve the student in making the puzzle first, and then in trying to solve it. This method emphasizes the importance of following instructions accurately, and introduces the element of group competition. Making classroom sets of some of the puzzles such as Tower of Hanoi or 10 Men in a Boat could serve as a good shop project for a student whose talents lie in that direction.

Some teachers leave a few of the games around the room on display so that students will see them and become interested. This allows students to attempt some of the puzzles before or after class and sometimes even before or after school. The attraction of a puzzle is frequently so great that a student will ask to check out the puzzle so that he can continue his search for a solution at home.

Although each one of the E S P's is completely self-explanatory, occasionally it will be advantageous to have one of them explained to the whole class. This is also a good way of orienting the class to the use and care of the puzzles. Each student should understand when and how to use these ideas. Such a presentation might be made more effective by having a student study a particular E S P and then demonstrate it to the class.

E S P's can be used to introduce a unit as might be done using the wire heart to lead into a study of topology. Or, it can be used to show the application of new learnings while in the midst of a unit, as could be done by applying base two in the solution of the Tower of Hanoi. And, it can even be used to conclude a unit as is the case with the dissected square which is to be used at

the end of a unit on Euclidean constructions. E S P can even be used as a make-up assignment for a student who has been absent.

Once the idea of E S P has been established in a class, encourage the students to keep on the look-out for other puzzles which might fit into the set.

In using E S P in the classroom there are a number of cautions which should be observed:

1) Don't give out solutions to any of the puzzles.

   A student will lose interest if he is told the solution.

2) Don't be afraid to include puzzles that you are not interested in or that you are unable to solve. Students' interests and abilities differ considerably.

3) Don't over-do it. It is possible to ruin any idea by using it too long or too often.

In the pages which follow, a number of Enrichment Student Projects have been collected, complete with solutions for the teacher's convenience. An effort has been made to be as complete as possible telling how to make it or where to buy it, giving the instructions which could be cut out and taped on an index card, and suggesting possible extensions to a whole class. Each section is numbered independently so that additions can be made conveniently.

II: PEG PUZZLES

1) TEN MEN IN A BOAT

2) REVERSAL

3) QUADRIX

4) ELIMINATION PLUS

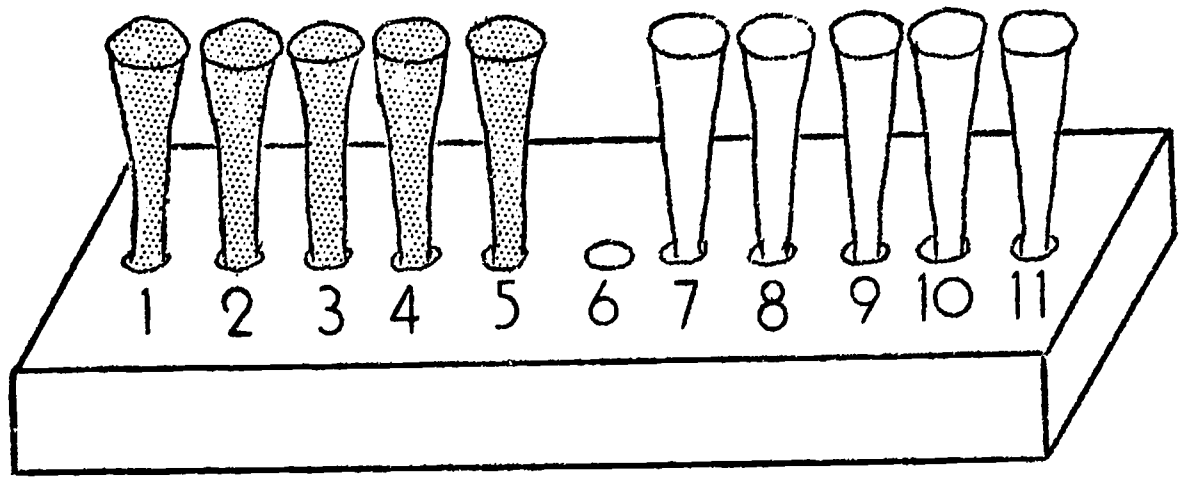
5) ELIMINATION SQUARE

6) ELIMINATION TRIANGLE

7) GARAGE SHUFFLE

All the puzzles in this section can be made on wood playing boards or on peg board, using golf tees for players. Their object is to discover a pattern which will lead to a solution.

TEN MEN IN A BOAT



Materials: A piece of wood 1" × 2" × 12", and ten golf tees, five of each of two different colors.

Preparation: Drill eleven evenly spaced holes in the wooden "boat" to accommodate the ten "men," five of one color at each end of the boat leaving the center "seat" empty. A light coat of sealer completes the project. Enclose the puzzle in a box with the cards below.

ESP TEN MEN IN A BOAT P - 1 A

Object: To reverse the position of the players in as few moves as possible.

Rules: 1) You may either jump a player as in checkers or move to an adjacent empty seat.

2) You may not jump a player of the same color.

Can you do it? How many moves does it take?

ESP Ten Men In A Boat P - 1 B

Add one more rule to those given on (A):

3) Players may not move backwards--each player may only move towards the opposite end from which they started.

Does this cut down the number of moves in your solution?

How many moves do you think are necessary now?

ESP Ten Men In A Boat P - 1
C

Put just two players on the board, one on each side of the center hole. How many moves are required to reverse their positions?

Try the same thing with four, six, and eight players. How many moves does it take for each?

Do you see a pattern?

Could you predict the number of moves it would take for twelve players?

Can you suggest a rule or formula that will tell the number of moves for any number of players?

Class Procedure:

1) Introduce the puzzle with a story like the following:

In a small midwestern city there were two exclusive schools, one for boys and one for girls. One day five of the boys decided to arrange blind dates with five girls at the other school.

The girls' school consented to the arrangement on one condition; that their chaperon, Miss Lina, must accompany the group. The girls, knowing Miss Lina's fear of water (she always got dreadfully sea sick), suggested a boat ride. When they arrived at the lake only one boat was available, a racing canoe seating eleven people single file. Since Miss Lina was unwilling to risk getting sick, she contented herself with supervising the seating arrangement, putting the girls at one end of the boat and the boys at the other. As quickly as the group was well out on the lake they decided to exchange seats to arrive at a more agreeable arrangement.

To make more of a challenge of the project, they decided on several rules: only one person can change seats at a time (for fear of upsetting the boat); it is permissible to move to an empty seat or to pass a fellow canoer, but no girl may pass another girl and no boy may pass another boy.

The object of the game is for all the canoers to move to the opposite end of the boat from which they started and to do this in as few moves as possible. How many moves will it take? Why would they decide on such a project? How are the canoers arranged at the half-way point?

2) Pass out the puzzles and allow the class sufficient time to experiment with the idea and to formulate a solution. (Note: If a set of wood boats cannot be made, coins or paper clips or other such markers could be used on a paper boat having eleven squares arranged in a row.)

3) If the minimum number of moves is not discovered, add one rule: you may never move backwards. If someone does come up with a solution, ask him to demonstrate to the class how it is done.

4) To analyze the puzzle, put the table below on the board or overhead projector and have the class fill in the information by experimenting. The correct answers are in parentheses.

# of Men	# of Moves
0	(0)
2	(3)
4	(8)
6	(15)
8	(24)
10	(35)
12	(48)
100	(2,600)
N	$(\frac{1}{4}N^2+N)$

Questions for class discussion:

If there were just two people in the boat, one on each side of the center seat, how many moves would it take for them to change seats?

How many for four? six? eight? (Make sure the class agrees with the facts before you put them in the table.)

Is there a pattern here?

Can you predict how many moves it would take to move twelve men? A hundred men?

Can you suggest a rule that will predict the number of moves for any number of men?

5) This puzzle does generate a quadratic equation of the form $Y = Ax^2 + Bx + C$. Substitution of the first set of facts shows that $C = 0$. Substitution of the next two sets gives us two equations in two unknowns. Solving for A and B simultaneously gives us the equation:

$$Y = \frac{1}{4} X^2 + X$$

where X equals the number of men and Y equals the number of moves required to reverse them.

6) Another pattern can be observed by counting the number of moves in each direction within a solution. For example with two men, you move one to the right into the empty hole, then you move the other to the left passing the first one, and finally one to the right to the empty seat, a total of three moves for one pair of players. Notice how the pattern developes.

Number of Men	Men moved in each direction	Total
1 pair	1 1 1	= 3
2 pairs	1 2 2 2 1	= 8
3 pairs	1 2 3 3 3 2 1	= 15
4 pairs	1 2 3 4 4 4 3 2 1	= 24
5 pairs	1 2 3 4 5 5 5 4 3 2 1	= 35
	R L R L R L R L R L R	

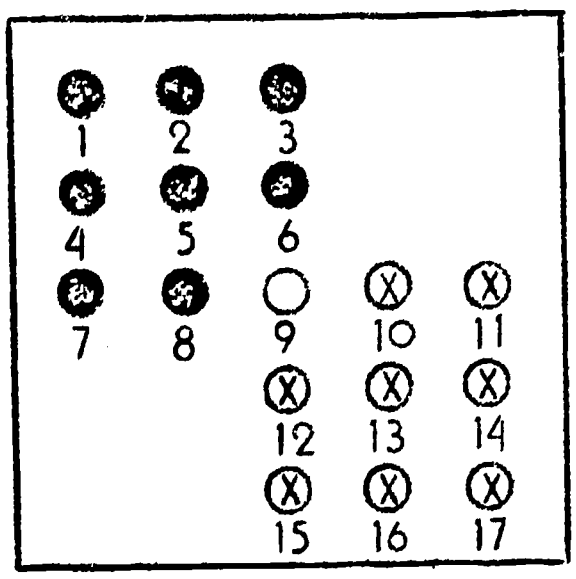
Solution: The steps of the solution for ten men is listed below. The first number stands for the position of the peg you are to move and the second number for the position to which you are to move it. (5 to 6, means move the peg in the fifth hole to the sixth hole.) Read down, from left to right.

A	B	C	D
5 to 6	10 to 9	9 to 7	7 to 5
7 to 5	8 to 10	11 to 9	9 to 7
8 to 7	6 to 8	10 to 11	8 to 9
6 to 8	4 to 6	8 to 10	6 to 8
4 to 6	2 to 4	6 to 8	4 to 6
3 to 4	1 to 2	4 to 6	5 to 4
5 to 3	3 to 1	2 to 4	7 to 5
7 to 5	5 to 3	3 to 2	6 to 7
9 to 7	7 to 5	5 to 3	

REVERSAL

Materials: A piece of wood 1" × 6" × 6" and sixteen golf tees, eight each of two different colors.

Preparation: Drill seventeen holes in the piece of wood as in the pattern opposite. Insert the tees, one color at the top and the other at the bottom, leaving the center hole vacant.



Enclose the puzzle in a box with the cards below.

ESP REVERSAL P - 2 A

Object: To reverse the position of the players so that all the bottom ones are moved to the top and vice versa.

Rules: 1) You may always move to an empty hole or jump a player of a different color.

2) You may never jump a player of the same color.

Can you do it? How many moves does it take?

ESP REVERSAL P - 2 B

Add one rule to those on (A):

3) The bottom players may move only to the left and up, while the top players may move only to the right and down. Neither set of players may move backwards.

Does this help you to reduce the number of moves you thought were necessary?

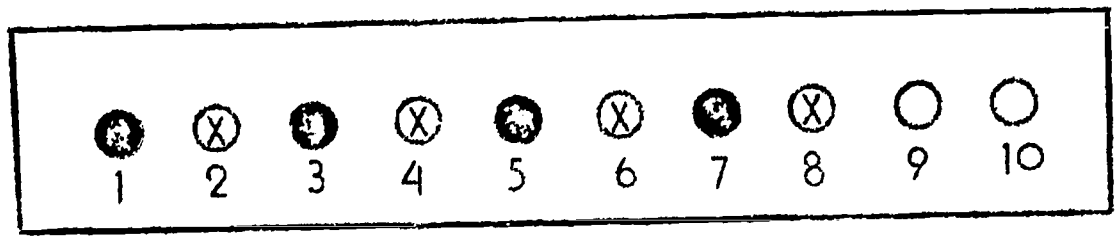
Source: Stancraft Products
1810 Como Avenue
St. Paul, Minnesota 55108

Game: FORE and AFT
Material: Walnut or Oak
Price: $5.00

olution: The first number indicates the position of the peg to be moved and the second to the position it is to be moved. REVERSAL may be done in forty-six (46) moves. Read down, left to right.

A	B	C	D
10 to 9	9 to 8	17 to 11	3 to 2
8 to 10	11 to 9	16 to 17	9 to 3
7 to 8	14 to 11	10 to 16	15 to 9
9 to 7	12 to 14	13 to 10	12 to 15
12 to 9	6 to 12	12 to 13	6 to 12
6 to 12	5 to 6	6 to 12	9 to 6
3 to 6	8 to 5	4 to 6	11 to 9
9 to 3	2 to 8	7 to 4	10 to 11
15 to 9	1 to 2	9 to 7	8 to 10
16 to 15	7 to 1	10 to 9	9 to 8
10 to 16	9 to 7	8 to 10	
8 to 10	11 to 9	2 to 8	

QUADRIX

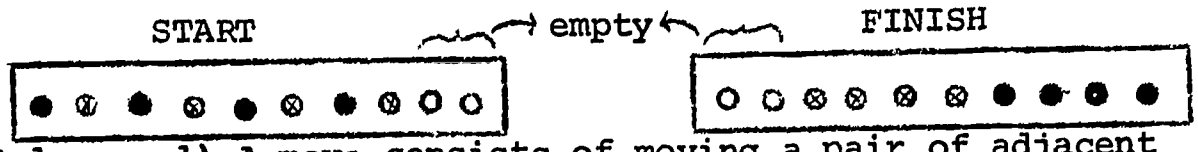


Materials: A piece of wood 1" × 2" × 12" and eight golf tees, four each of two different colors.

Preparation: Drill ten holes in the wood as in the pattern above. Insert the two sets of tees in an alternating pattern leaving the right hand pair of holes vacant.

ESP QUADRIX P - 3

Object: To go from START to FINISH in just four moves.



Rules:
1) A move consists of moving a pair of adjacent players, regardless of color, to a pair of adjacent empty holes.

2) You can pass any number of players moving in either direction

3) You may not reverse the order of the pair or separate the pair.

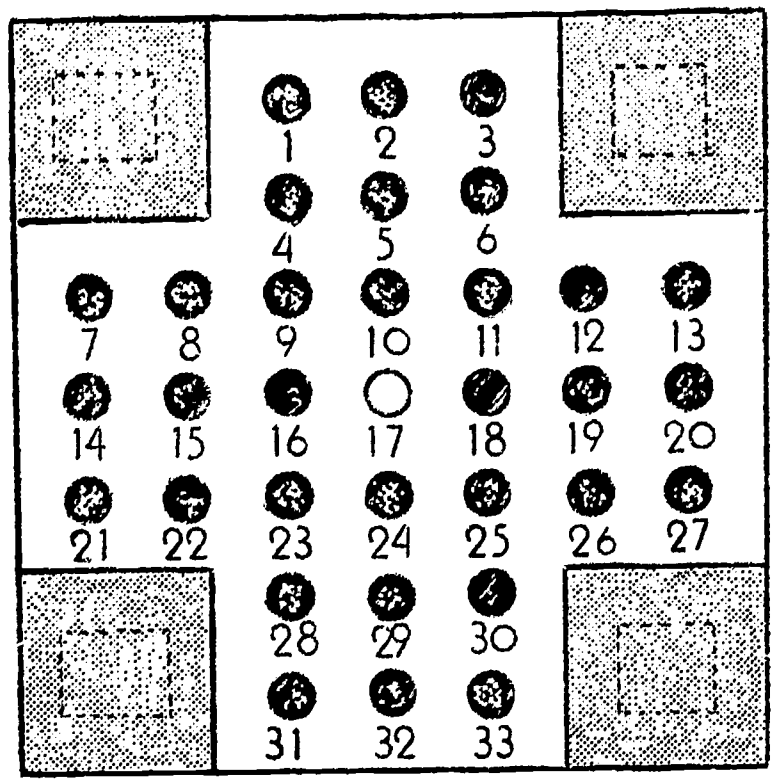
Solution: The first pair of numbers indicates the pegs to move, and the second pair, the holes to which they are to be moved.

2 - 3 to 9 - 10
5 - 6 to 2 - 3
8 - 9 to 5 - 6
1 - 2 to 8 - 9

ELIMINATION PLUS

Materials: A piece of peg board seven holes square and four 1" square blocks or a piece of wood 1' × 8" × 8". Also, 32 golf tees of the same color.

Preparation: Cover or paint the unwanted sections of peg board and glue the blocks on each corner of the bottom to act as legs.
Or, if you prefer wood, drill the holes as in the pattern opposite.
Insert the tees in all except the center hole.



ESP ELIMINATION PLUS P - 4

Object: To eliminate all but one player ending with that player in the center hole.

Rules:
1) Beginning with the center hole empty, jump horizontally and vertically removing the jumped players as in checkers.

2) Every move must be a jump.

3) You may not jump diagonally.

How many can you eliminate? Can you get down to just one? How many moves does it take?

Sources:

Kohner Bros. Inc.
155 Wooster Street
New York, New York 10012

Game: Hi Q
Material: Plastic
Cost: $1.00

Stancraft Products
1810 Como Avenue
St. Paul, Minnesota 55108

Game: CROSS
Material: Walnut or Oak
Cost: $5.00

Solution: The first number gives the position of the peg to be moved and the second number gives the position to which it is to be moved, removing the peg jumped each time.

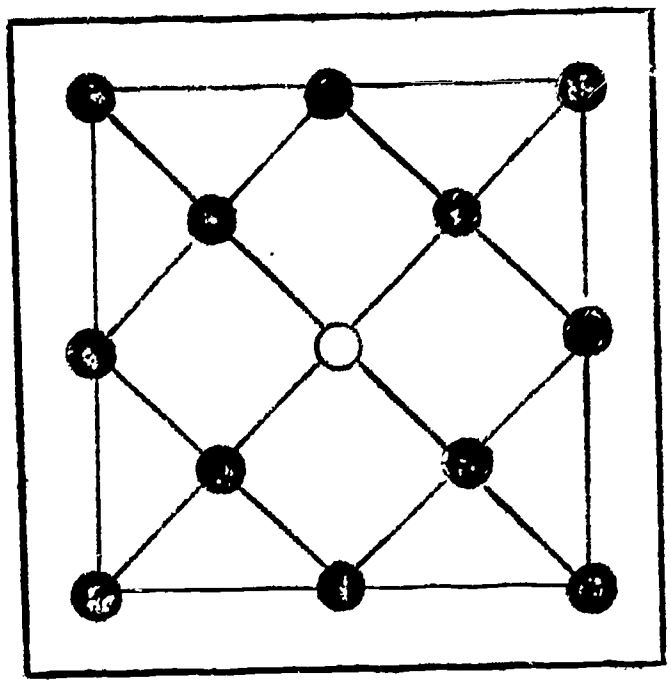
ELIMINATION PLUS can be done in thirty-one (31) jumps. This is only one of the many possible solutions. Read down, beginning with column A.

A	B	C	D
29 to 17	31 to 23	16 to 4	13 to 11
26 to 24	24 to 22	7 to 9	10 to 8
33 to 25	21 to 23	18 to 30	11 to 3
17 to 29	16 to 28	20 to 18	3 to 1
32 to 24	14 to 16	11 to 25	1 to 9
24 to 26	9 to 23	30 to 18	8 to 10
27 to 25	28 to 16	3 to 11	5 to 17
22 to 24	1 to 9	18 to 6	

ELIMINATION SQUARE

Materials: A piece of wood 1" × 6" × 6" and twelve golf tees of the same color.

Preparation: Drill thirteen holes in the wood as in the pattern at right. Insert the golf tees leaving the center hole vacant.



ESP — ELIMINATION SQUARE — P - 5

Object: To eliminate all but one of the players ending with the last player in the center hole.

Rules:
1) Jump as you would in checkers, removing the jumped players.
2) You may jump in any direction provided the holes are adjacent and in a straight line.

Can you do it? How many moves did it take?

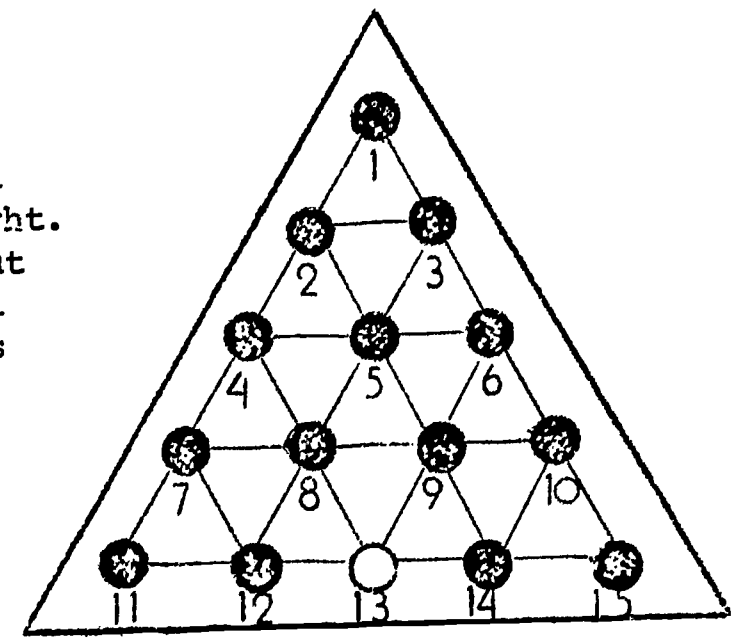
Source: Stancraft Products
1810 Como Avenue
St. Paul, Minnesota 55108

Game: Bull's-Eye
Material: Walnut or Oak
Price: $5.00

ELIMINATION TRIANGLE

Material: A piece of wood 1" × 6" × 6" and fourteen (14) golf tees of the same color.

Preparation: Drill fifteen holes in the wood according to the pattern at right. The game looks better if you cut the wood to make an equilateral triangle. Insert the golf tees leaving the center hole on the bottom row vacant. Variations are made possible by leaving other holes vacant at the start of the game. The most popular variation is to leave a corner hole vacant.



ESP ELIMINATION TRIANGLE P - 6

Object: To eliminate all but one of the players, ending with the last player in the hole which was vacant at the start of the game.

Rules: 1) Jump as you would in checkers, removing the jumped players.

2) You may jump in any direction, provided the holes are adjacent (next to each other) and in a straight line.

How many players can you eliminate? How many moves did it take you?

Source: Stancraft Products
1810 Como Avenue
St. Paul, Minnesota 55108

Game: Tri
Material: Walnut or Oak
Price: $5.00

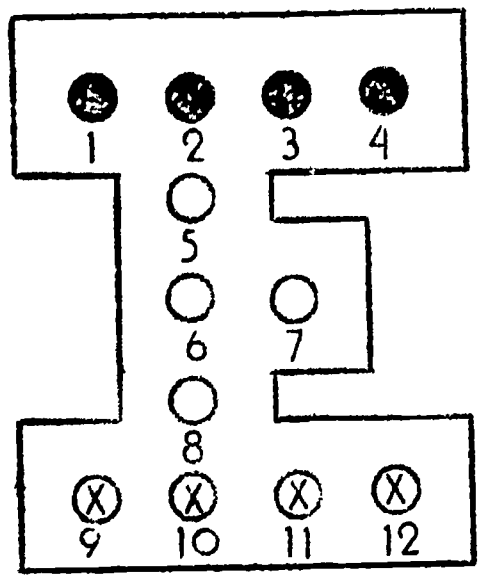
Solution: The first number gives the position of the player you are to move, the second number gives the position of the hole to which you are to move.

A	B	C
6 to 13	7 to 2	13 to 6
2 to 9	1 to 4	10 to 3
15 to 6	12 to 5	3 to 8
3 to 10	14 to 12	4 to 13
	11 to 13	

GARAGE SHUFFLE

Material: A piece of wood 1" × 6" × 6" or a piece of pegboard and eight golf tees, four each of two different colors.

Preparation: Drill the holes in the wood as in the pattern opposite. If using pegboard, paint or cover the unused holes. Insert four golf tees of the same color across the top of the "E," the remaining four across the bottom, leaving the center four holes empty.



ESP GARAGE SHUFFLE P - 7

Object: To switch the cars from one end of the "E" shaped garage to the other in the least number of moves possible.

Rules:
1) All paths are one lane, so it is impossible to pass another car.
2) You may move only one car at a time, but you may move in any direction, provided the way is clear.
3) Each car may be moved several times before arriving at its final parking spot.

Can you reverse the position of the cars? How many moves does it take?

Solution: The first number gives the position of the car you are to move, the second number gives the position to which you are to move. The change is possible in at least thirty-five moves.

A	B	C	D	E
2 to 7	8 to 11	4 to 5	7 to 11	7 to 9
10 to 2	7 to 9	1 to 4	5 to 10	5 to 10
9 to 5	5 to 10	5 to 1	2 to 8	2 to 8
7 to 9	2 to 7	6 to 3	3 to 7	1 to 7
11 to 7	1 to 8	8 to 7	8 to 3	8 to 1
12 to 8	7 to 1	10 to 2	10 to 2	10 to 2
9 to 12	3 to 6	11 to 5	9 to 5	7 to 10

III: DISSECTION PUZZLES

1) TANAGRAMS

2) DISSECTED SQUARE

3) DISSECTED "T"

4) CROSS DISSECTIONS

5) STAR TRICK

6) BUTTERFLY PUZZLE

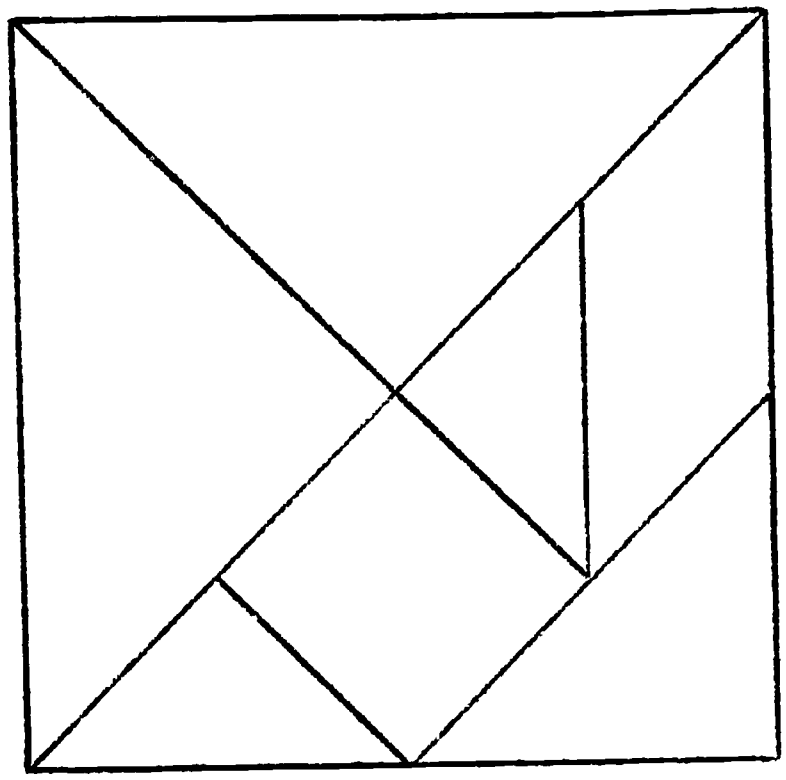
7) HEXED

8) VOODOO

Dissection puzzles are made out of cardboard or wood and their object is to reconstruct the original figure. Due to the similarity of pieces, more than one combination is possible. These are only a few of the more popular dissection puzzles, but they should serve as an introduction to puzzles of this type.

TANAGRAMS

Preparation: Draw the pattern opposite on a piece of cardboard 5" × 5" and cut out the pieces with a razor blade or scissors.

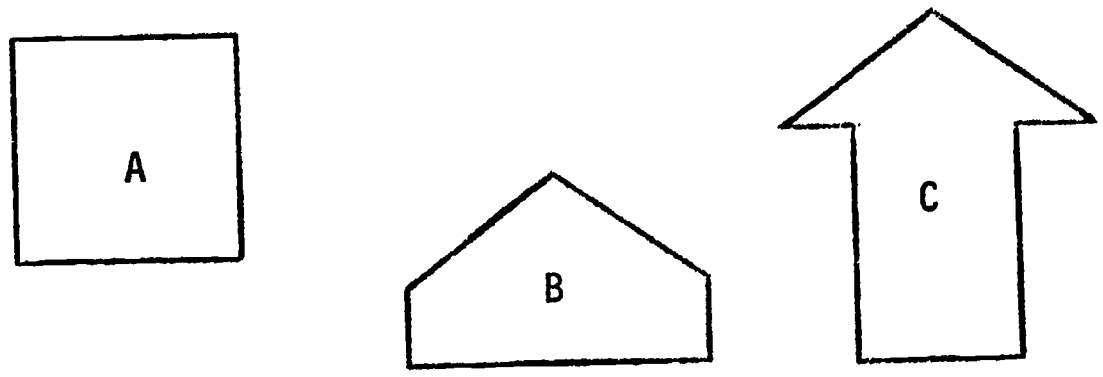


Class Procedure: The puzzle is easily constructed and analyzed by a whole class. For instructions, work sheets, and a set of silhouettes refer to:

TANAGRAMS @ $.75
from: CILAMP
1164 - 26th Street
Des Moines, Iowa
50311

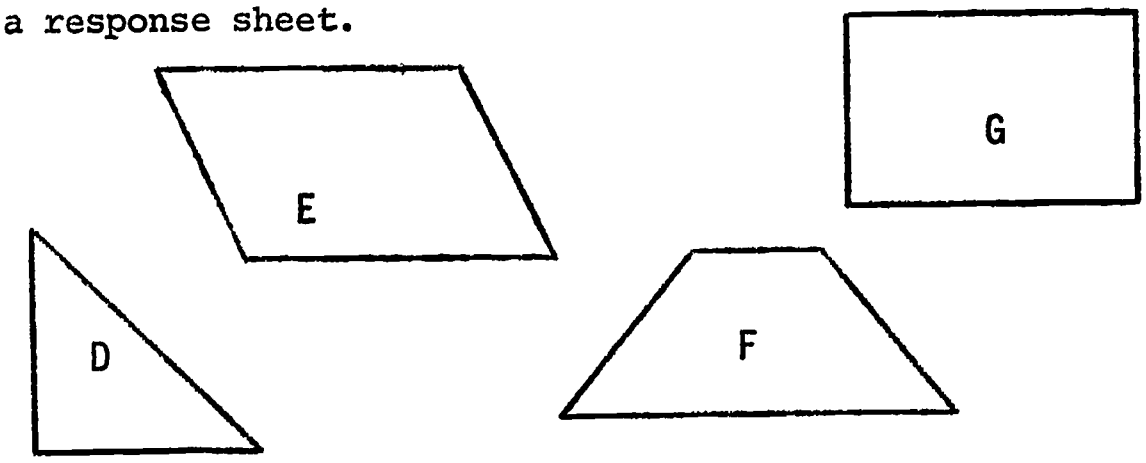
ESP TANAGRAMS D - 1
A

These seven geometric pieces, if arranged correctly, can be used to make a variety of different figures including the ones below. Each figure requires the use of all seven tanagram pieces. Can you make these? Sketch your solutions on a response sheet.



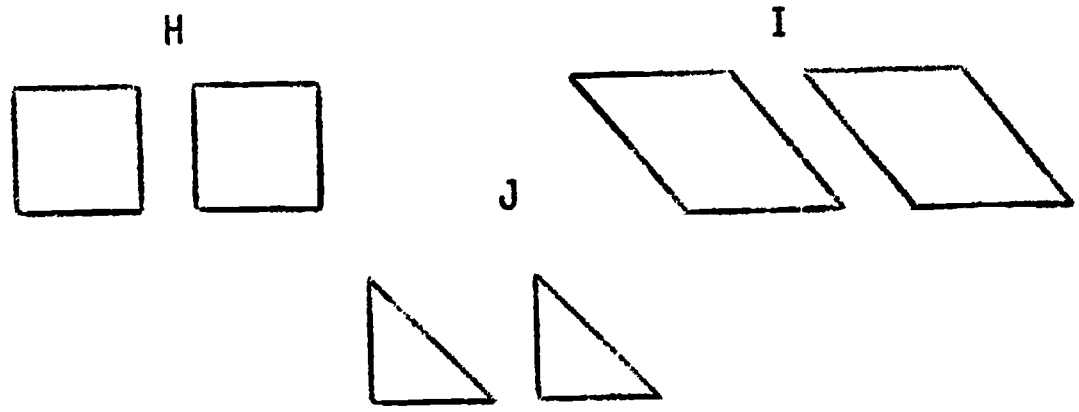
ESP TANAGRAMS D - 1 B

The four figures below are interesting because each can be made from any other by moving just one piece. Each one uses all seven tanagram pieces. Sketch your solutions on a response sheet.

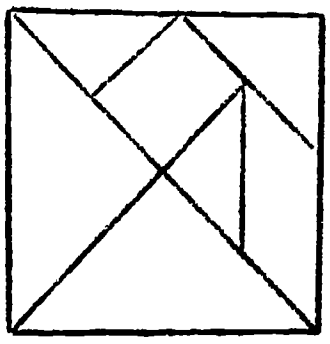


ESP TANAGRAMS D - 1 C

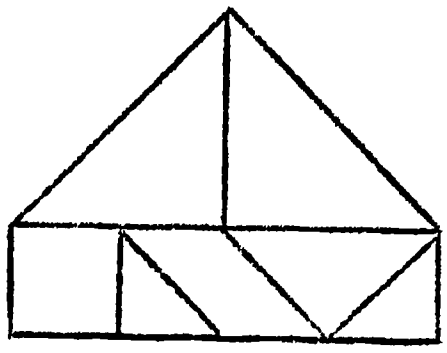
It is also possible to make pairs of congruent (identical) figures using all seven pieces. Are all the ones below possible? Sketch your solutions on a response sheet.



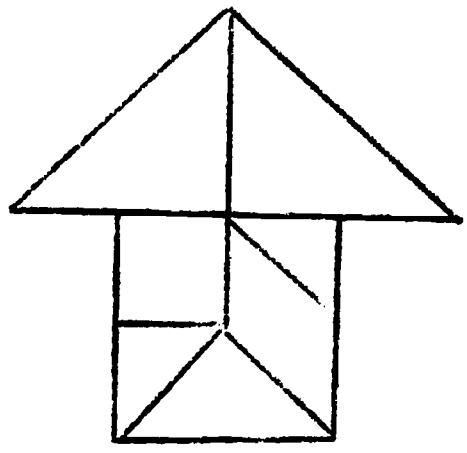
Solutions:



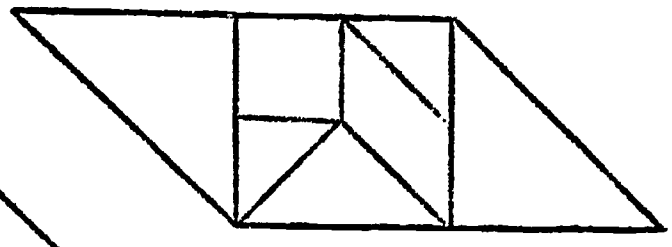
A



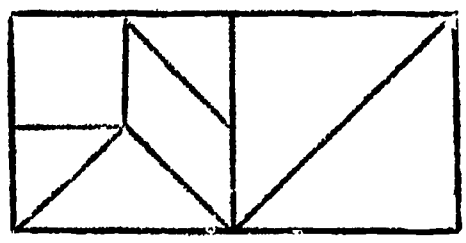
B



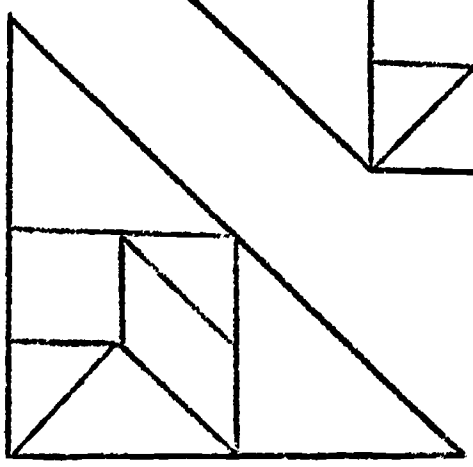
C



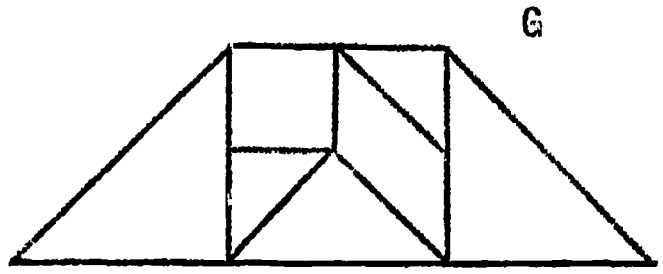
E



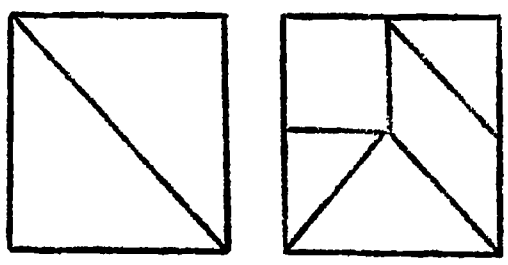
G



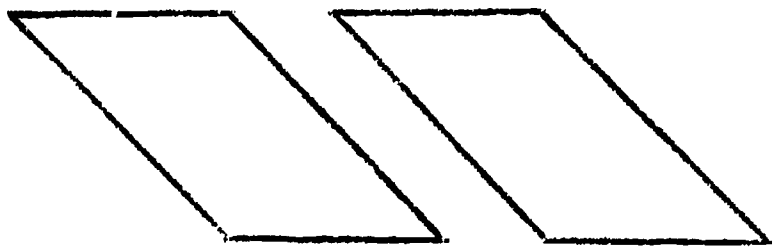
D



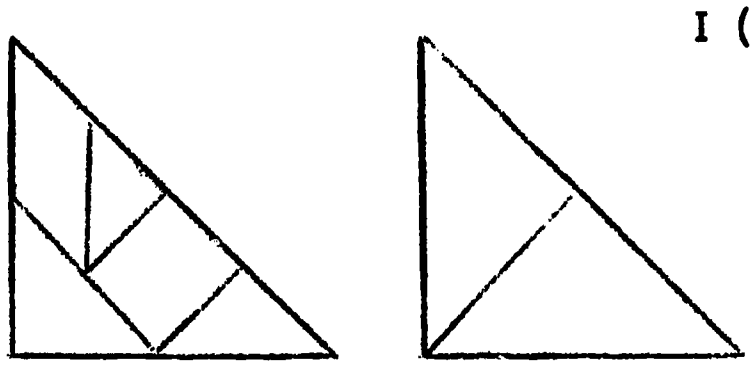
F



H



I (impossible)



J

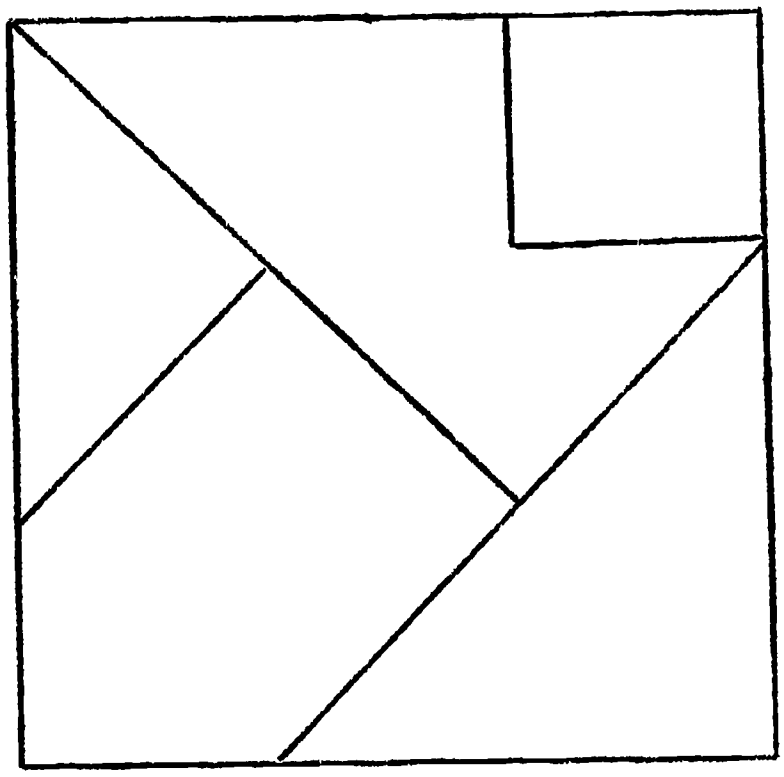
Source: Kohner Brothers Inc.
155 Wooster Street
New York, New York 10012

Game: Pythagoras
Material: Plastic puzzle and leaflet of silhouettes
Price: $1.00

Reference: Tangrams, Ronald C. Read, Dover, 1965.

DISSECTED SQUARE

Preparation: Draw the pattern at right on a piece of cardboard and cut out the pieces using a razor blade or scissors.



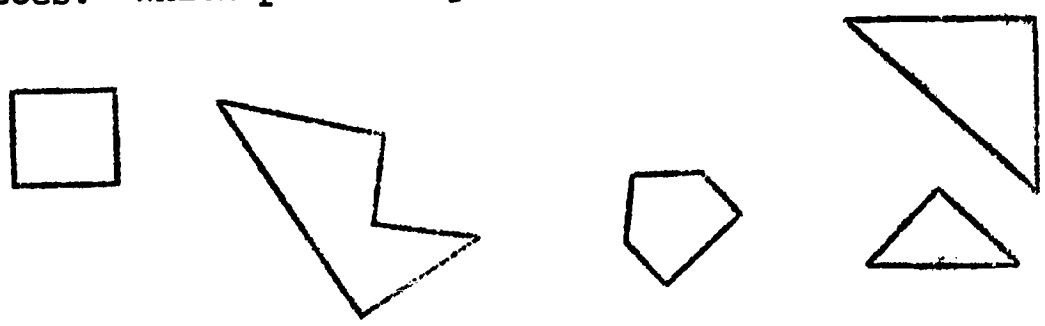
Project: Have a student make up a set of instructions for making the puzzle using just pencil and ruler, or, perhaps, compass and straight edge (Euclidean constructions). Then, test the instructions by having the whole class follow them and when finished, suggested how they might be improved. If you prefer to use something which is ready to use, duplicate the instruction sheet (following page).

NOTE: These instructions presume that the student has learned the basic Euclidean constructions: a) bisecting a line, b) bisecting an angle, and c) constructing a perpendicular. The puzzle is a good way of concluding a unit on constructions.

ESP DISSECTED SQUARE D - 2

These five pieces can be used to make a square. Every piece must lie flat on your desk and none of the pieces may be placed on top of any of the others. Can you do it? Sketch your solution on a response sheet.

It is also possible to make a square using just four of the pieces. Which piece do you have to leave out?



PUZZLING CONSTRUCTION

Using Euclidean construction procedures, it is possible to make a puzzle composed of five separate pieces. Before you perform the constructions, read through the directions and make a sketch of each piece to decide where each should be placed on a single sheet of paper. After you have finished the constructions, cut the pieces out and see if you can make a square using all five pieces.

Figure A:
1) Draw $\overline{BC}$ approximately two inches in length.
2) Construct $\overline{AB} \perp \overline{BC}$ and $\overline{AB} = \overline{BC}$.
3) Draw $\overline{AC}$.

Figure B:
1) Draw $\overline{ST}$ so that it is twice the length of $\overline{AB}$.
2) Bisect $\overline{ST}$ and label the midpoint as X.
3) Construct $\overline{RS} \perp \overline{ST}$ and $\overline{RS} = \overline{ST}$.
4) Bisect $\overline{RS}$ and label the midpoint as Y.
5) Draw $\overline{RT}$ and $\overline{XY}$.
6) Bisect $\overline{RT}$ labeling the midpoint as Z.
7) Bisect $\overline{XY}$ labeling the midpoint as W.
8) Draw $\overline{WZ}$.

Figure C:
1) Construct $\overline{LM} = \overline{WX}$.
2) Construct $\overline{OL} \perp \overline{LM}$ and $\overline{NM} \perp \overline{LM}$ such that both O and N are on the same side of $\overline{LM}$ and that $\overline{OL} = \overline{LM} = \overline{MN}$.
3) Draw $\overline{ON}$.

Figure D:
1) Construct $\overline{EF} = \overline{AC}$.
2) Construct $\overline{GE} \perp \overline{EF}$ and $\overline{GE} = \overline{EF}$.
3) Draw $\overline{GF}$.

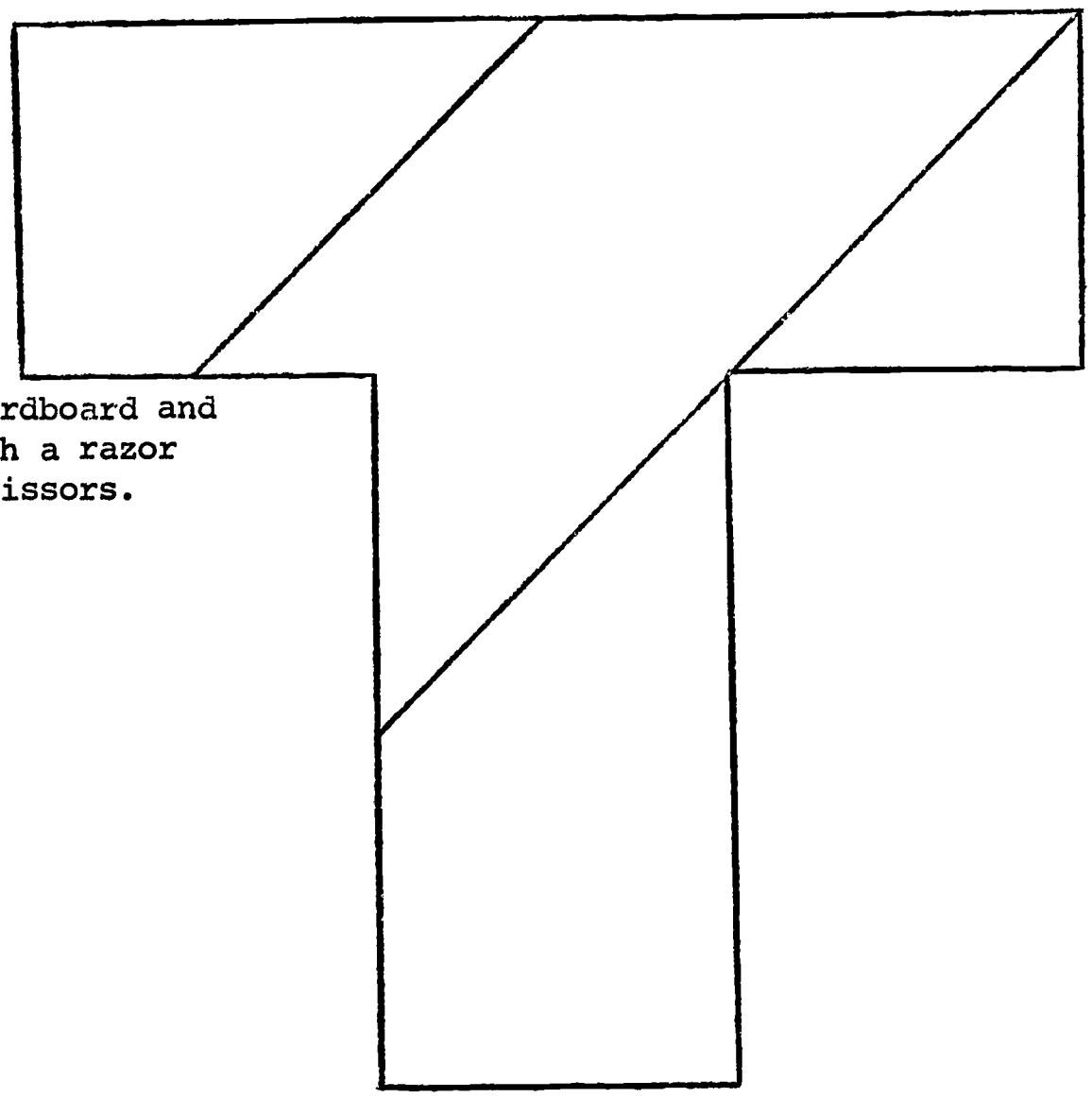
Figure E:
1) Construct $\overline{HI} = \overline{LM}$.
2) Construct $\overline{JH} \perp \overline{HI}$ and $\overline{JH} = \overline{HI}$.
3) Draw $\overline{JI}$. (Use a dotted line--do NOT cut.)
4) Construct $\overline{FJ} \perp \overline{JI}$ and $\overline{FJ} = \overline{AB}$.
5) Construct $\overline{KI} \perp \overline{IJ}$ and $\overline{KI} = \overline{JF}$.
6) Draw $\overline{FK}$.

Cut out triangle ABC, pentagon RSXWZ, quadrilateral LMNO, triangle EFG, and pentagon HIKFJ. See if you can make a square using all five pieces.

DISSECTED "T"

Material: A piece of cardboard about 5" × 5".

Preparation: Draw the pattern opposite on a piece of cardboard and cut out with a razor blade or scissors.

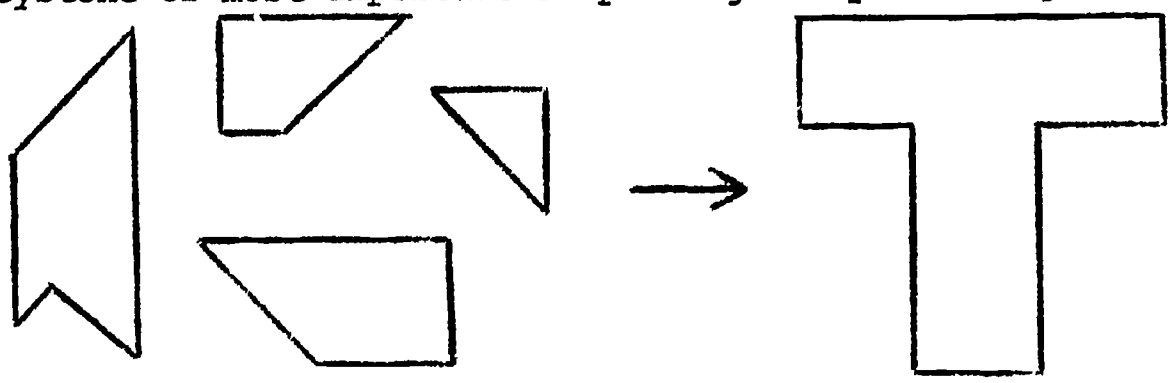


ESP DISSECTED "T" D - 3

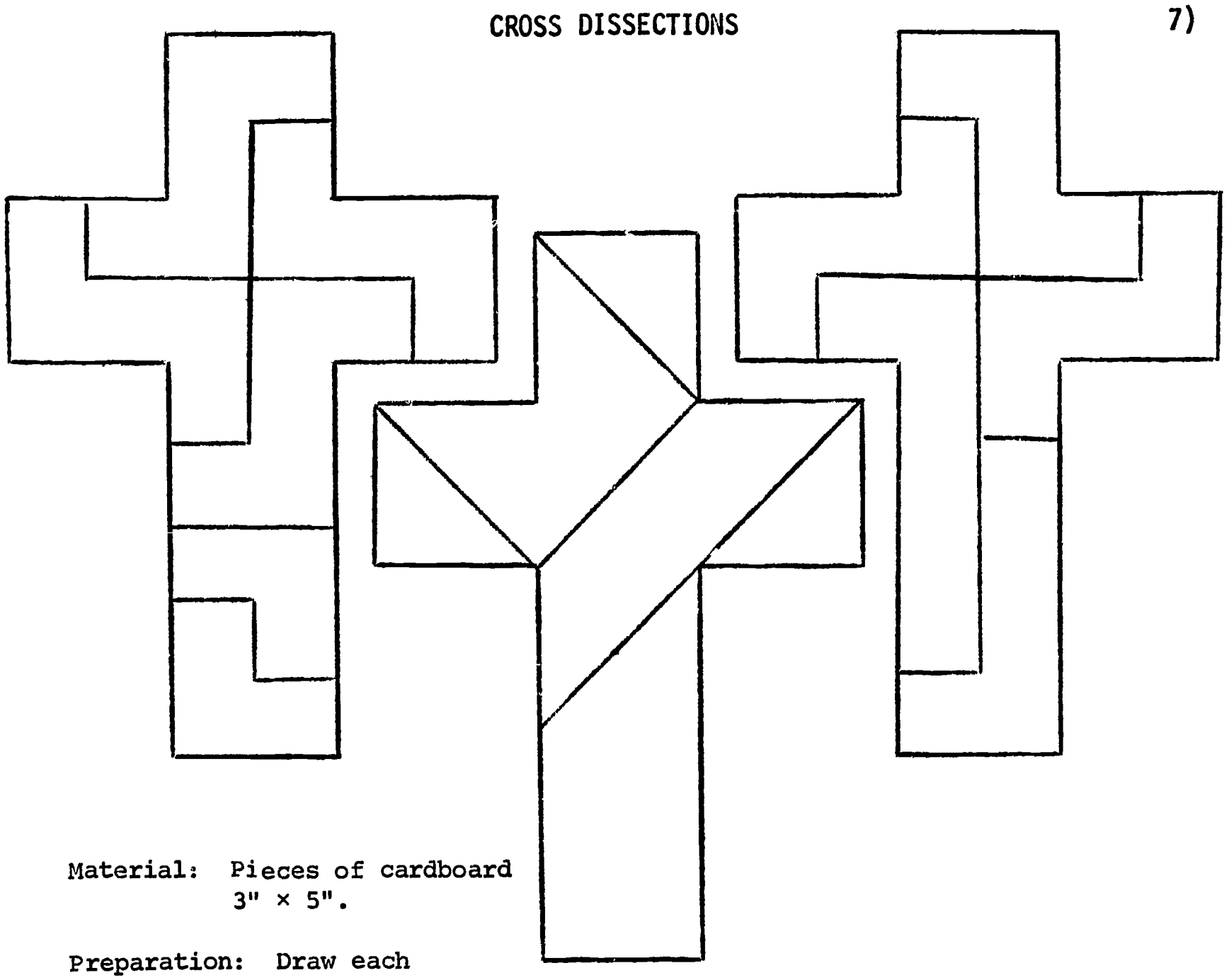
These four pieces can be used to form the letter "T." Can you do it? Sketch your solution on a response sheet.

Why was it difficult to make a "T" out of these pieces?

Place an X on your sketch to indicate which piece was the keystone or most important in putting the puzzle together.



CROSS DISSECTIONS



Material: Pieces of cardboard 3" × 5".

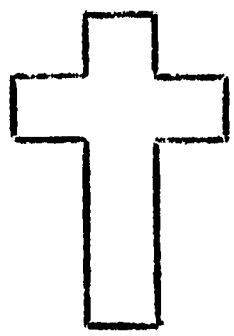
Preparation: Draw each pattern on a piece of cardboard, and cut out with a razor blade or scissors. Each pattern is a separate puzzle.

ESP CROSS DISSECTION D - 4

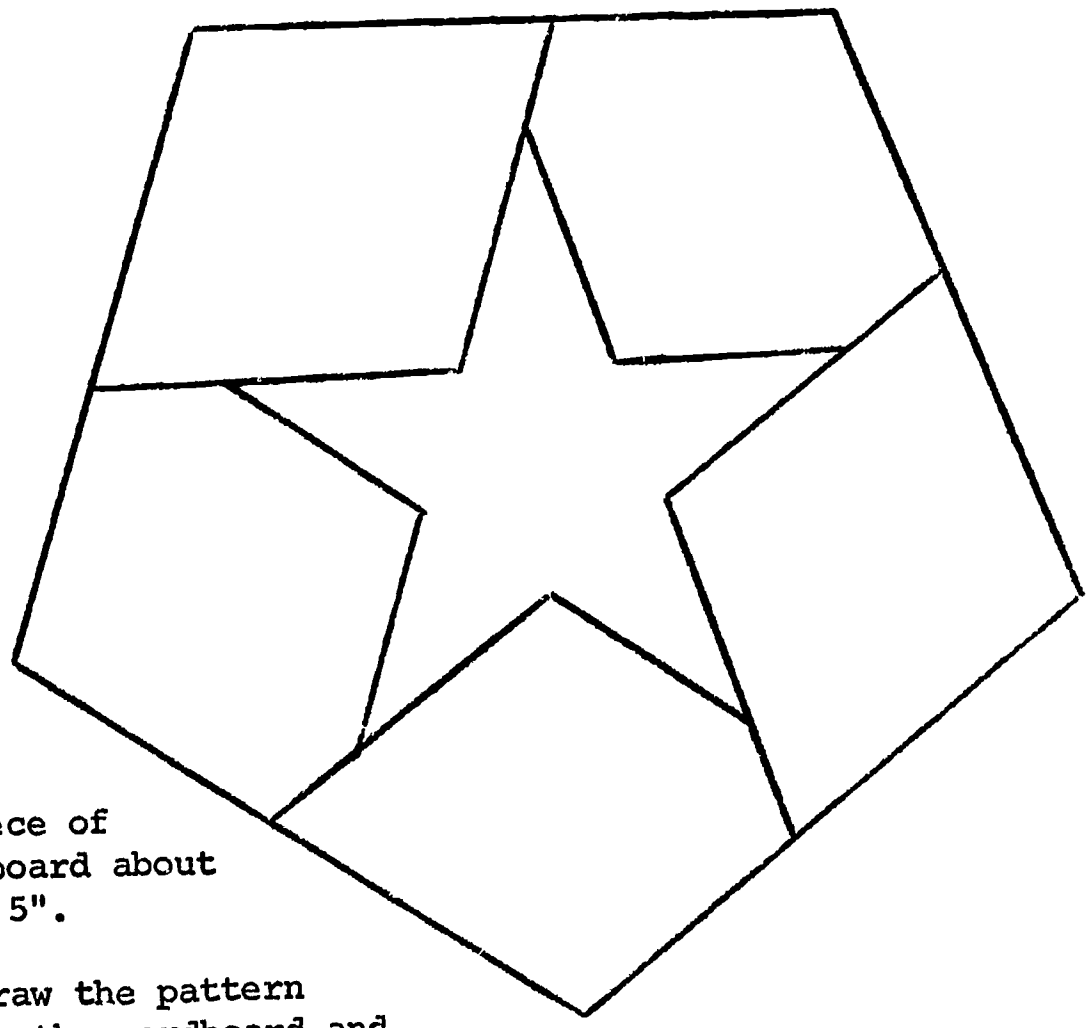
The pieces to this puzzle can be used to make a regular cross.

Can you do it?

Sketch your solution on a response sheet.



STAR TRICK



Material: A piece of cardboard about 5" × 5".

Preparation: Draw the pattern on the cardboard and cut out the five pieces, discarding the star.

ESP

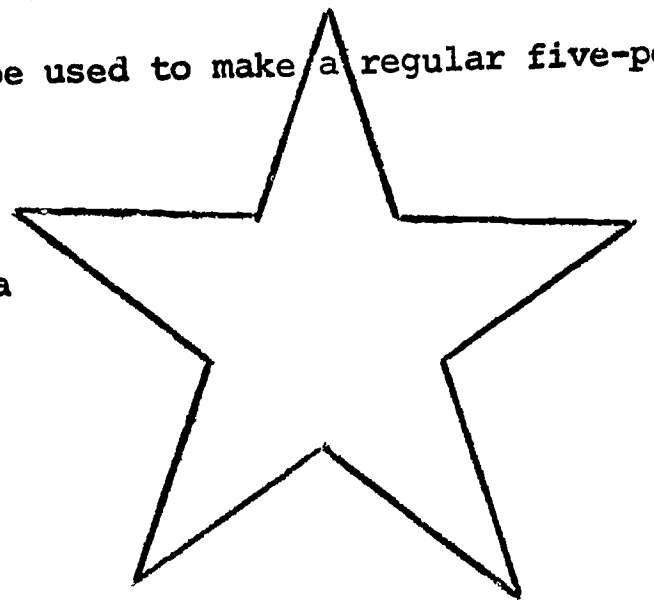
STAR TRICK

D - 5

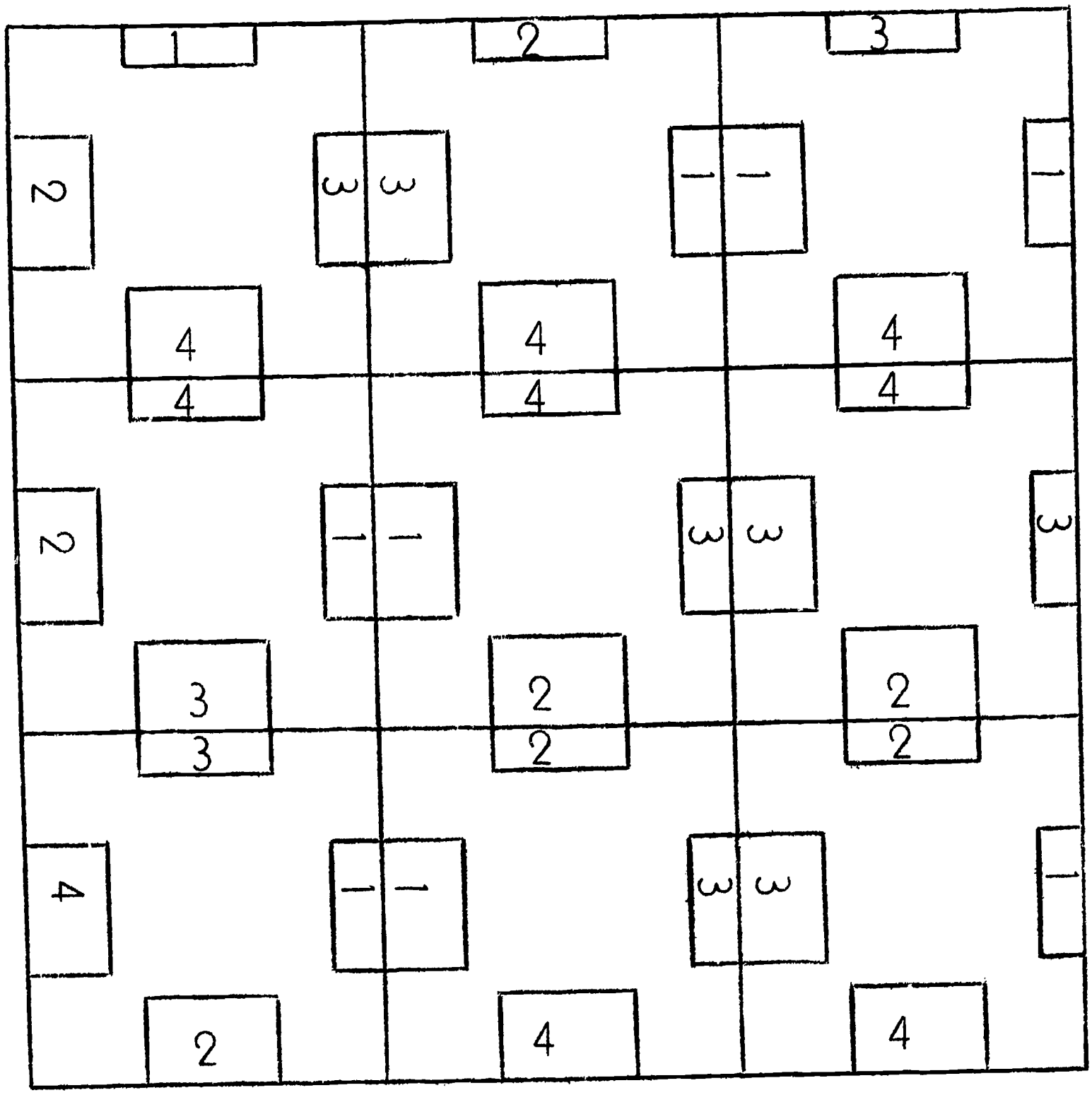
These five pieces can be used to make a regular five-point star.

What's the trick?

Sketch your answer on a response sheet.



BUTTERFLY PUZZLE



Material: Pieces of oak tag 6" × 6".

Class Procedure:

1) Pass out the pre-cut oak tag squares, rulers, and scissors.

2) By measuring, divide the square into nine equal squares, three rows of three squares each using the full piece of cardboard.

3) Put the pattern on the board or make an overlay from the pattern and put it on the overhead, and explain the process of locating the small squares. Each little square is 3/4" × 3/4" and each is located so that 2/3 of it is above the line and 1/3 is below the line in one direction and centered in the other. Centering the squares is an excellent problem in measurement. This phase is most important, since failure to center the squares will make finding a solution very difficult.

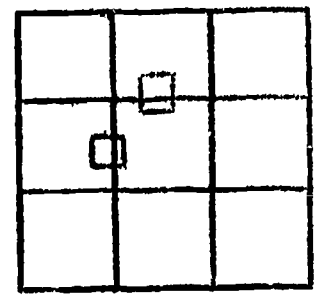
4) Number the squares as in the pattern, or replace each number by an original design or a different color.

5) Cut out the nine squares and see if you can reconstruct the puzzle.

Note: The name BUTTERFLY PUZZLE is due to its original form; the squares having been butterflies of various colors, each having a head and a tail. Since butterflies are rather difficult to draw, they were replaced by squares, circles or triangles. Creative individuals are invited to return the puzzles to its original form or any new form which might seem appealing.

ESP BUTTERFLY PUZZLE D - 6

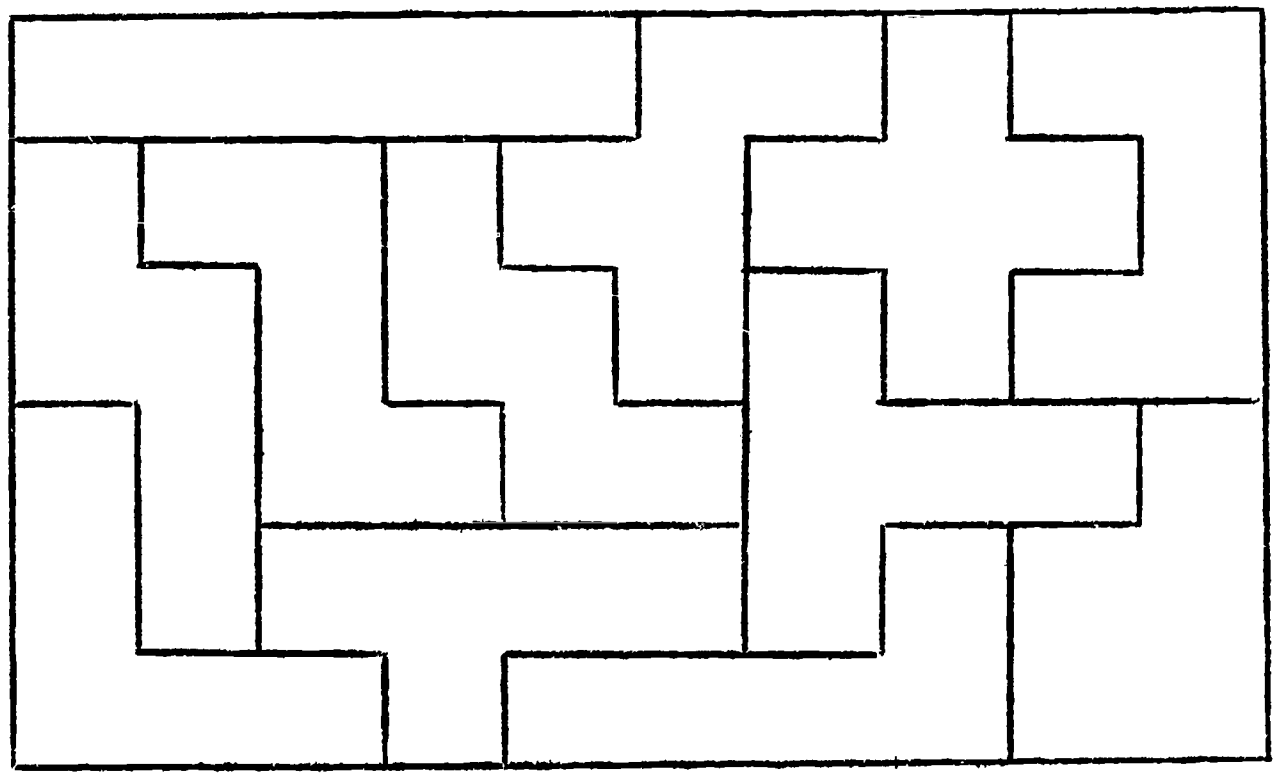
These nine squares can be arranged in a 3 × 3 square, such that each of the figures in the interior is a little square having matching numbers.



More than one solution is possible.

Sketch your solution on a response sheet.

Material: Plastic



ESP HEXED D - 7

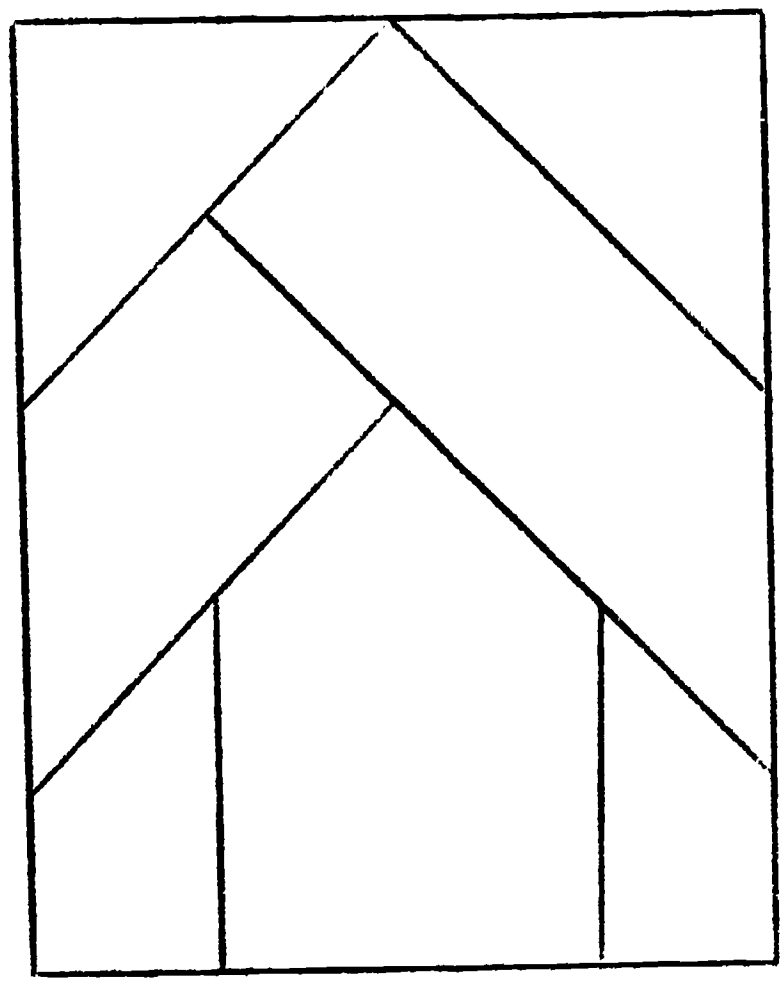
Object: To arrange the twelve pieces (each composed of five square units) into a rectangle either 6" × 10" or 5" × 12". A three dimensional version can be made from wood blocks, making possible a 3" × 4" × 5" solid.

Note: The three dimensional version is much too difficult for most puzzlists.

Source: Kohner Brothers Inc.
155 Wooster Street
New York, N.Y. 10012

Game: Hexed
Material: Plastic
Price: $1.00

Material: Plastic



ESP VOODOO D - 8

Object: To use the seven pieces to make a rectangle and 140 other geometric shapes.

Sources: Kohner Brothers Inc.
155 Wooster Street
New York, N.Y. 10012

Game: Voodoo
Material: Plastic
Price: $1.00

E. S. Low Company, Inc.
200 - 5th Avenue
New York, N.Y. 10010

Game: SHEKO
Material: Plastic
Price: $.49

IV: CUBE PUZZLES

1) SOMA CUBE

2) COMBO

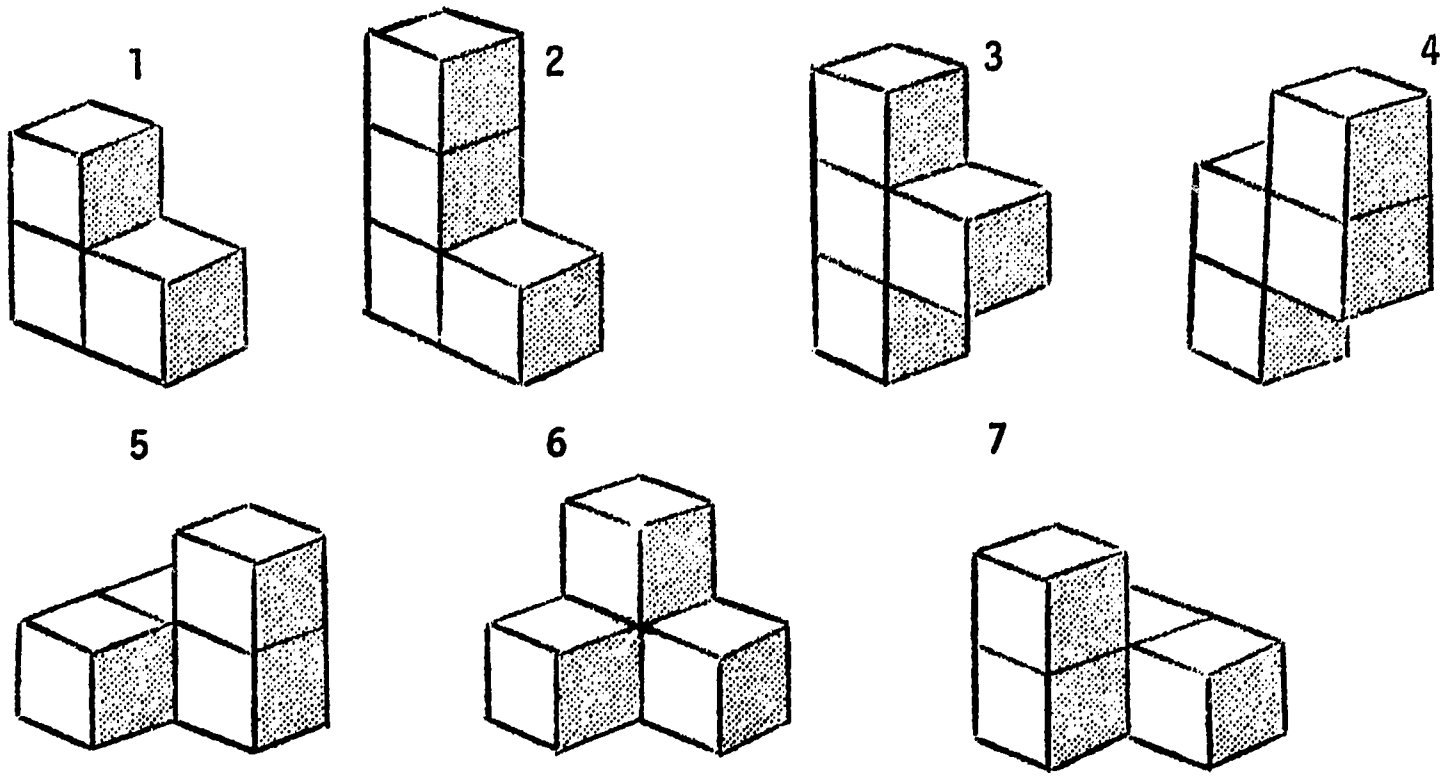
3) GAMBLER'S DIE

4) PAPER CUBE

5) ORIENTAL CUBE

All of these puzzles involve one or more cubes. Each requires for its solution a sense of form perception and an understanding of how the pieces are related.

SOMA CUBE



Material: Twenty-seven one inch cubes or a set of children's building blocks.

Preparation: Glue the blocks together as in the pattern above.

ESP

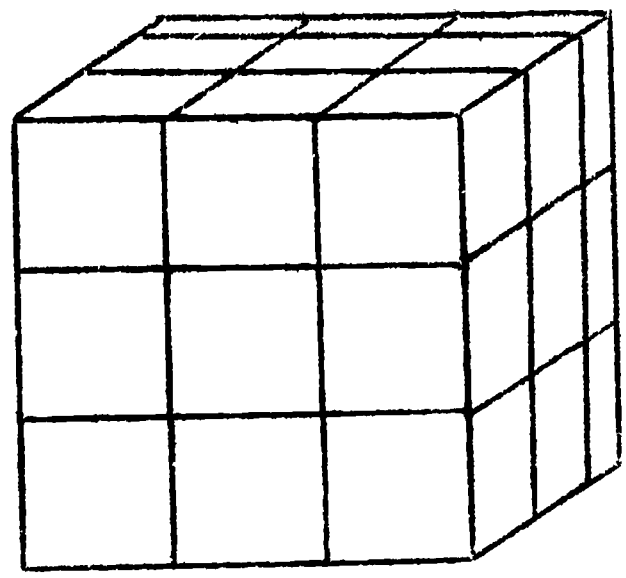
SOMA CUBE

C - 1
A

These seven pieces can be used to make a cube 3 × 3 × 3. There are supposedly over two hundred solutions to this puzzle.

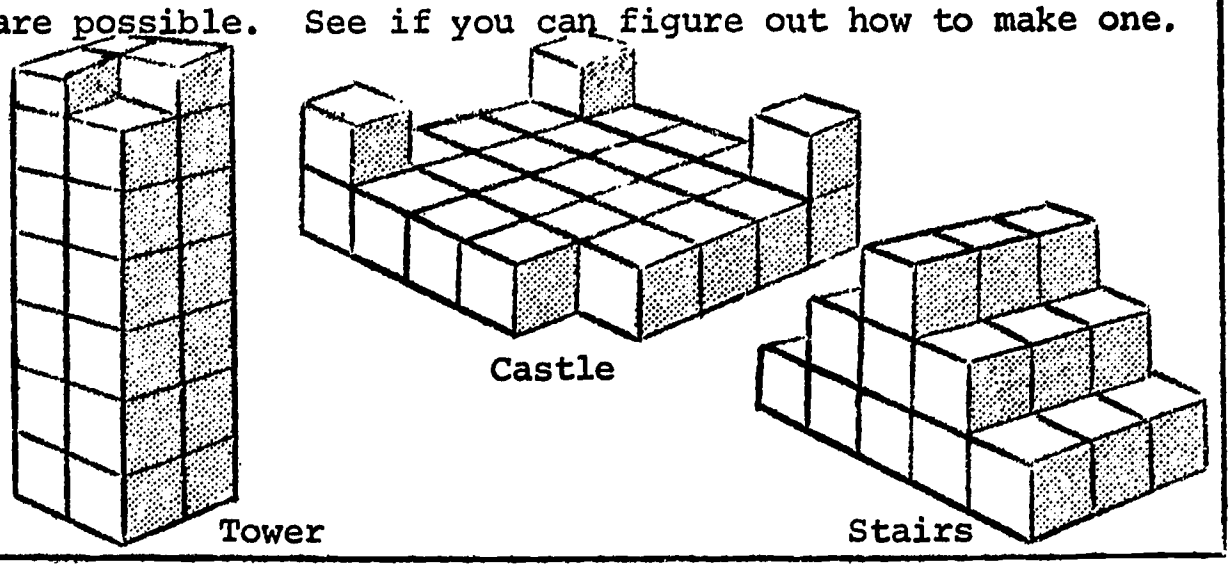
Can you find just one?

Sketch the position of each piece in your solution on a response sheet.

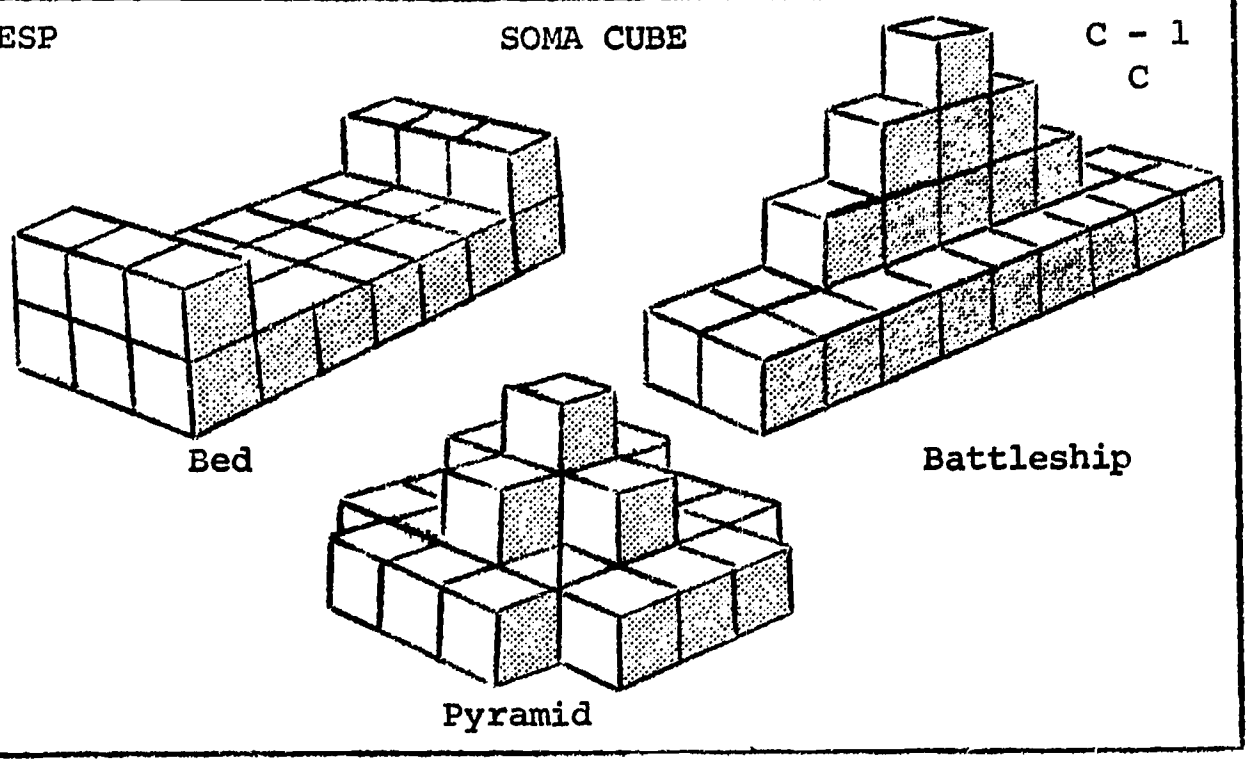


ESP SOMA CUBE C - 1
B

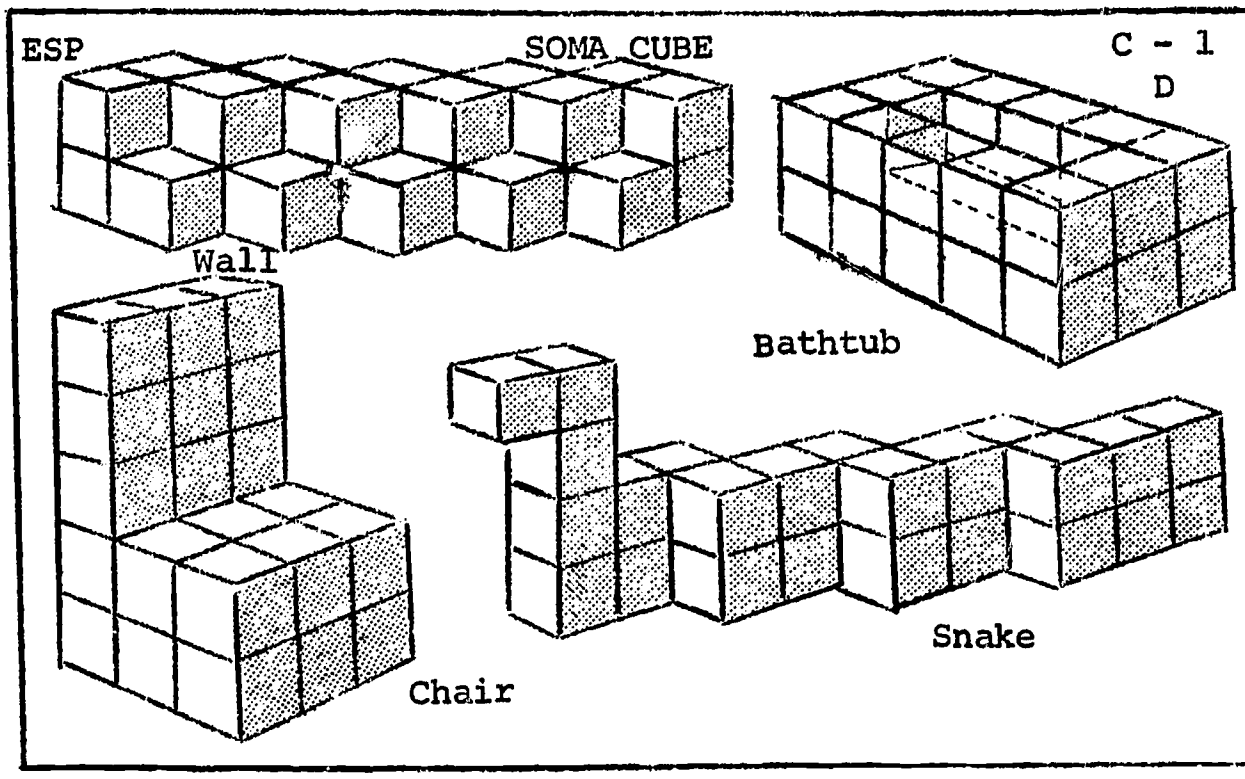
It is possible to make many other figures besides the cube. Some are harder than others, but all the ones listed below are possible. See if you can figure out how to make one.



ESP SOMA CUBE C - 1
C



ESP SOMA CUBE C - 1
D

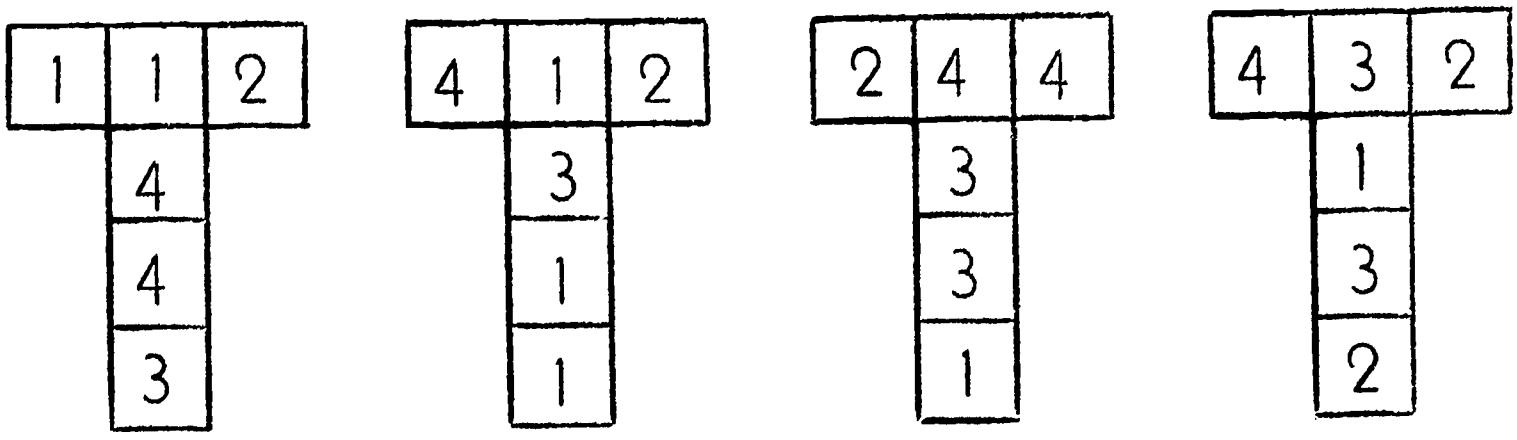


Source: Edmund Scientific Company
300 Edscorp Building
Barrington, New Jersey
08007

Game: Soma Cube
Material: Wood
Price: $3.00

Reference: The 2nd Scientific American Book of Mathematical Puzzles and Diversions, Martin Gardner, Simon and Schuster, New York, 1961.

COMBO



Material: Four plain wood blocks.

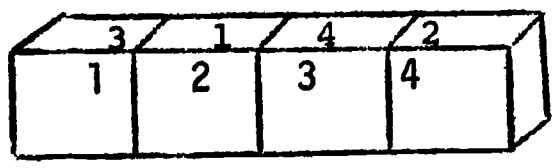
Preparation: Number the faces of the cubes as in the pattern above or replace each number by a color.

ESP COMBO C - 2

It is possible to arrange these four blocks in a row so that the digits 1, 2, 3, and 4 appear just once on each face. The digits may be in any order, for example: 3, 1, 2, 4 is a perfectly acceptable arrangement.

Can you do it?

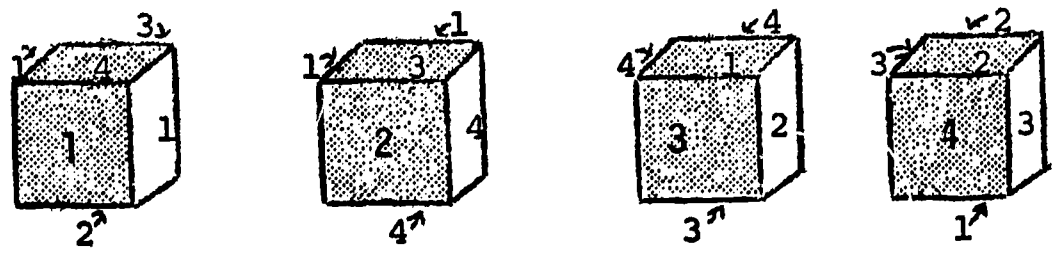
Sketch your solution on a response sheet, indicating which numbers are on the inside, bottom, back, etc.



Source: Parker Brothers
10215 Douglas Avenue
Des Moines, Iowa 50322

Game: Instant Insanity
Material: Plastic
Price: $1.00

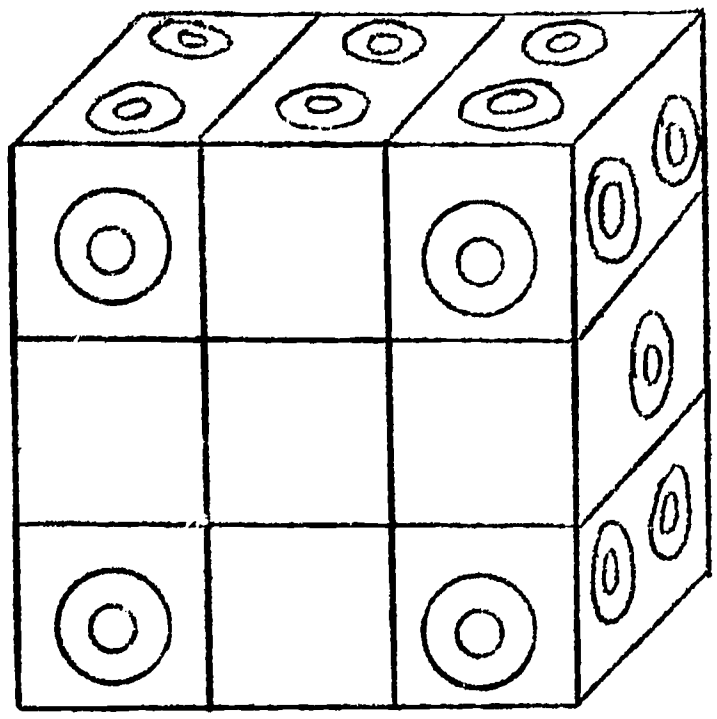
Solution:



GAMBLER'S DIE

Material: Nine blocks of wood, 3/4" × 3/4" × 2 1/4" and twenty-one notebook ring reinforcements.

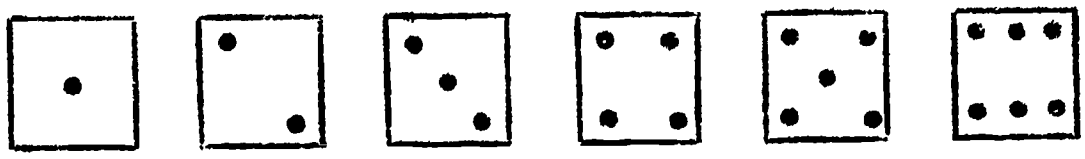
Preparation: Paint the blocks red or some other bright color and then stack to make a cube. Glue the circles on the faces and ends of the blocks as in the pattern opposite, or paint white dots instead of the reinforcements.



ESP GAMBLER'S DIE C - 3

These nine blocks can be arranged to make a regular gambler's die (singular of dice). The dots must appear in the correct pattern on each face as shown below. To be a correct solution, opposite faces must also add up to seven.

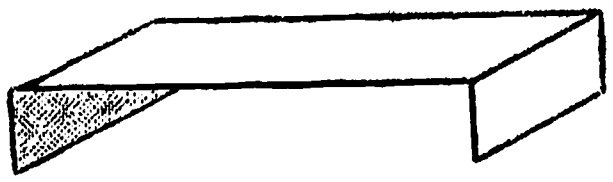
Can you do it?

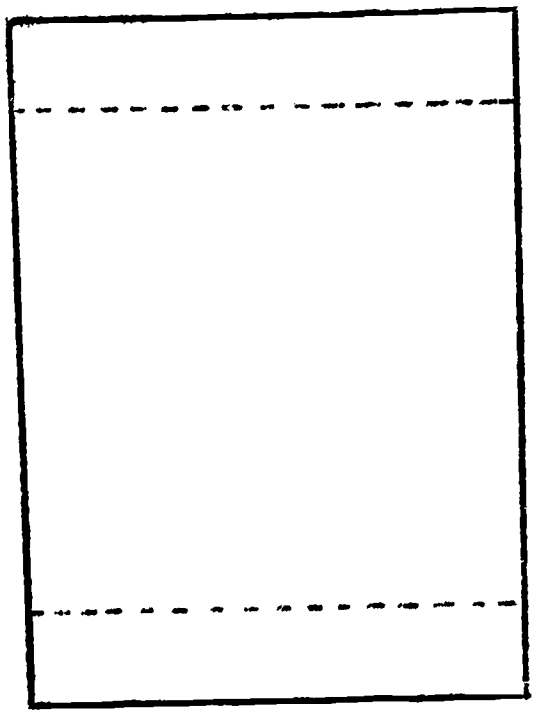


PAPER CUBE

Material: Regular notebook paper or white plain paper.

Preparation: Draw six rectangles, each 2" × 2 3/4". Draw in the 3/8" flaps on each end. Cut out the rectangles and fold the flaps at a 90 degree angle as shown below.

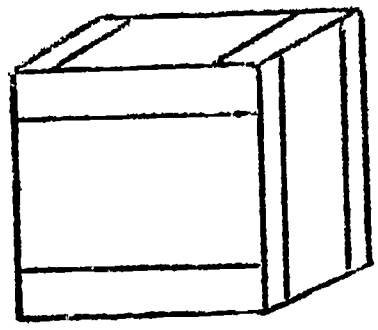




ESP PAPER CUBE C - 4

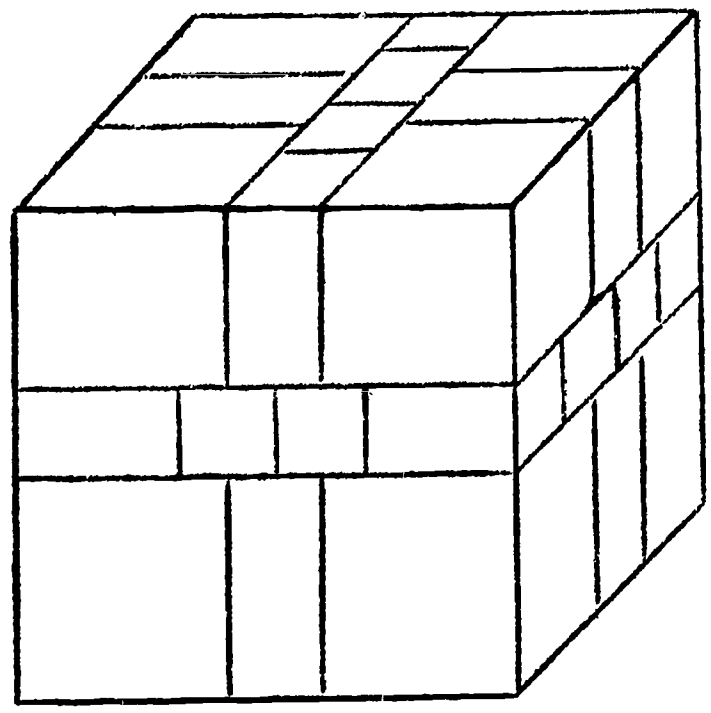
Using the six pieces of paper, it is possible to build a strong paper cube. No tape or glue is allowed or necessary. Can you discover how to put the pieces together? Do the flaps serve a useful purpose?

Make sure the flaps are bent at right angles to the square and in the same direction.



ORIENTAL CUBE

This is just one of many such puzzles which are available today. Among the more popular ones are:
a barrel, a pig, a mouse, an elephant, a sphere, a truncated cube, etc.



ESP ORIENTAL CUBE C - 5

Object: To discover how to take the cube apart and to remember how to put it back together.

Source: Edmund Scientific Co.
300 Edscorp Building
Barrington, New Jersey
08007

Game: Wood Puzzles
Price: $3.50

V: TOPOLOGY PUZZLES

1) WIRE HEART

2) SHOESTRING PUZZLE

3) KITE KNOT

4) THE GORDIAN KNOT

5) ANOTHER KNOT

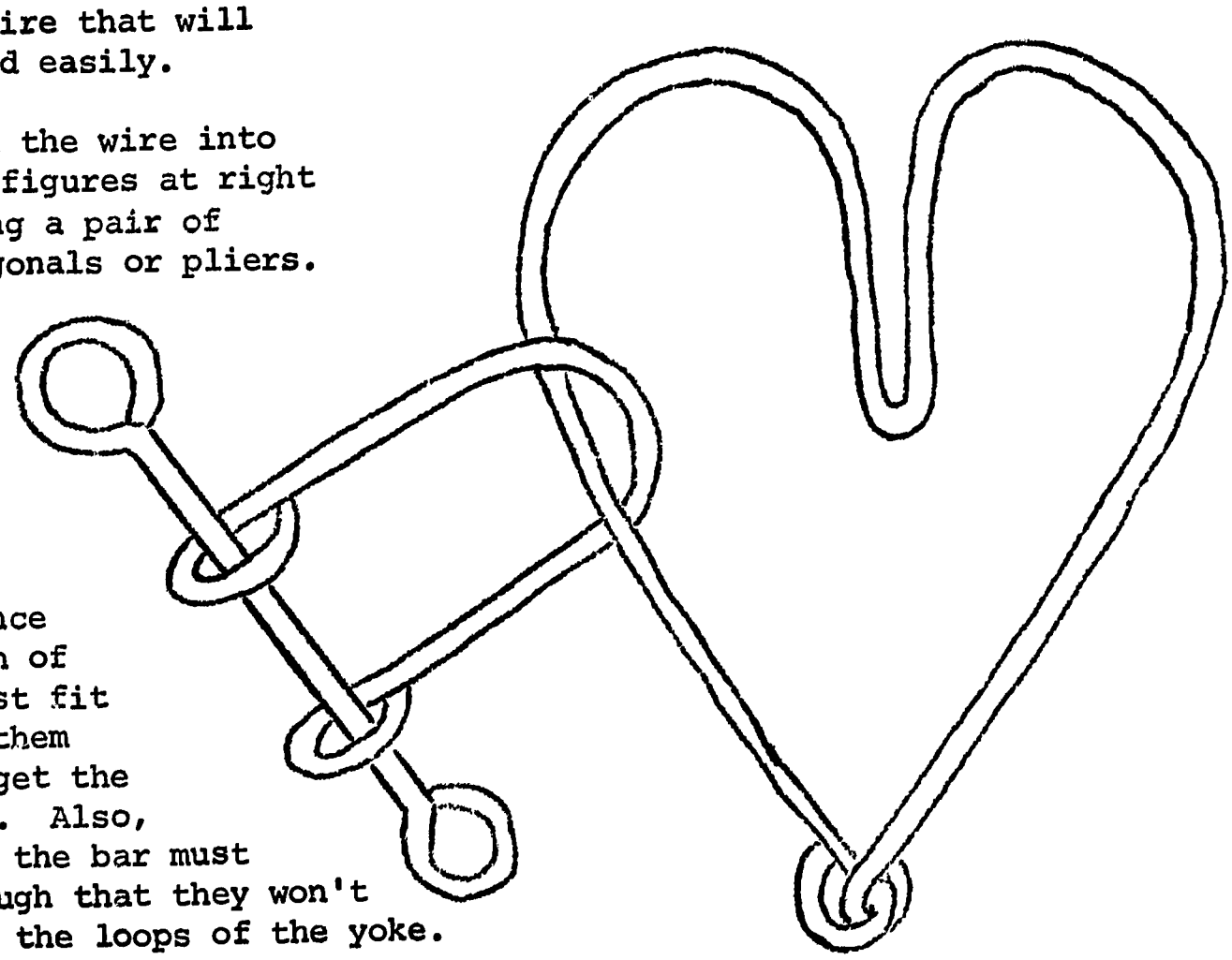
6) RING RELEASE

The puzzles in this section are all topological in nature. Each appears to have an inside and an outside, where as in fact there is just an outside. The solution of one of the puzzles simplifies the solution of the others, since they all work on pretty much the same principle. These puzzles could serve as an appetizer for other areas of topology such as networks, moebius strips, four-color maps, or flexigons.

WIRE HEART

Material: Heavy wire that will not bend easily.

Preparation: Bend the wire into the figures at right using a pair of diagonals or pliers.



Pay special attention to the size of the loops on the yoke, since the indention of the heart must fit into one of them in order to get the puzzle apart. Also, the loops on the bar must be large enough that they won't pass through the loops of the yoke.

ESP

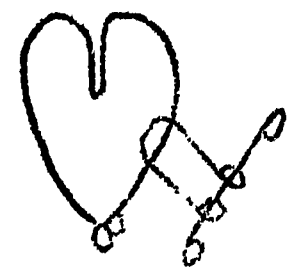
WIRE HEART

T - 1

It is possible to remove the yoke and bar from the wire heart without bending, forcing, or taking any joints apart.

Can you do it?

If so, describe your solution on a response sheet.



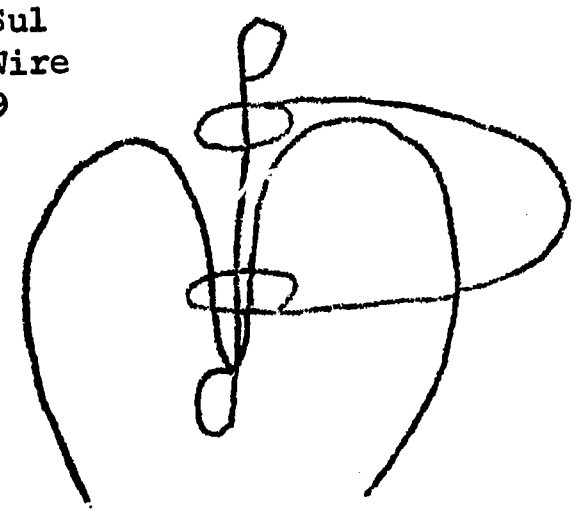
Before

After

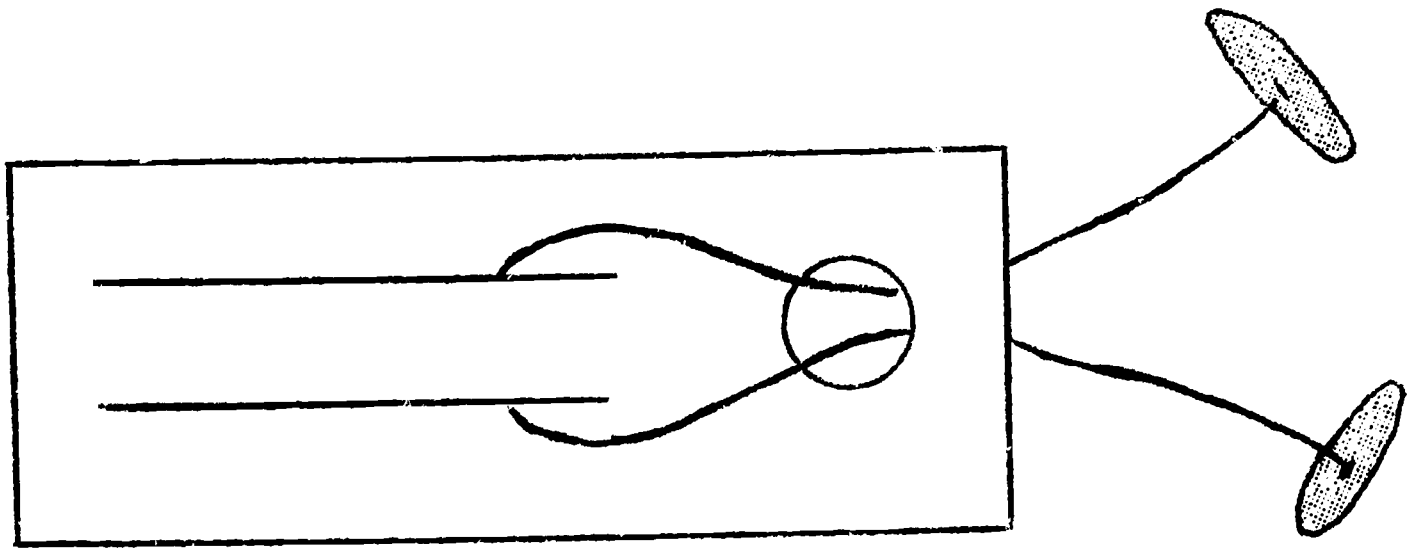
Source: B & L Gadgets
1421 - 5th Street, N. W.
Cedar Rapids, Iowa 52405

Game: Tus Sul
Material: Wire
Price: $.59

Solution: Insert the indention of the heart into one of the loops of the yoke as in the diagram and move the bar from one side of the indentation to the other.



SHOESTRING TROUBLE



Material: A piece of leatherette (upholstery material) about 3" × 5", a piece of string or leather shoe lace about 12" long and two buttons, poker chips, or metal washers.

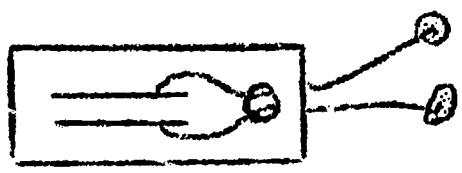
Preparation: Cut two slits and a hole in the leatherette making the hole slightly larger than the distance between the two slits. Attach the buttons to the ends of the shoe lace and the puzzle is complete.

ESP SHOESTRING TROUBLE T - 2

It is possible to remove the string from the piece of leatherette without untieing the buttons or forcing the buttons through the hole or cutting the string.

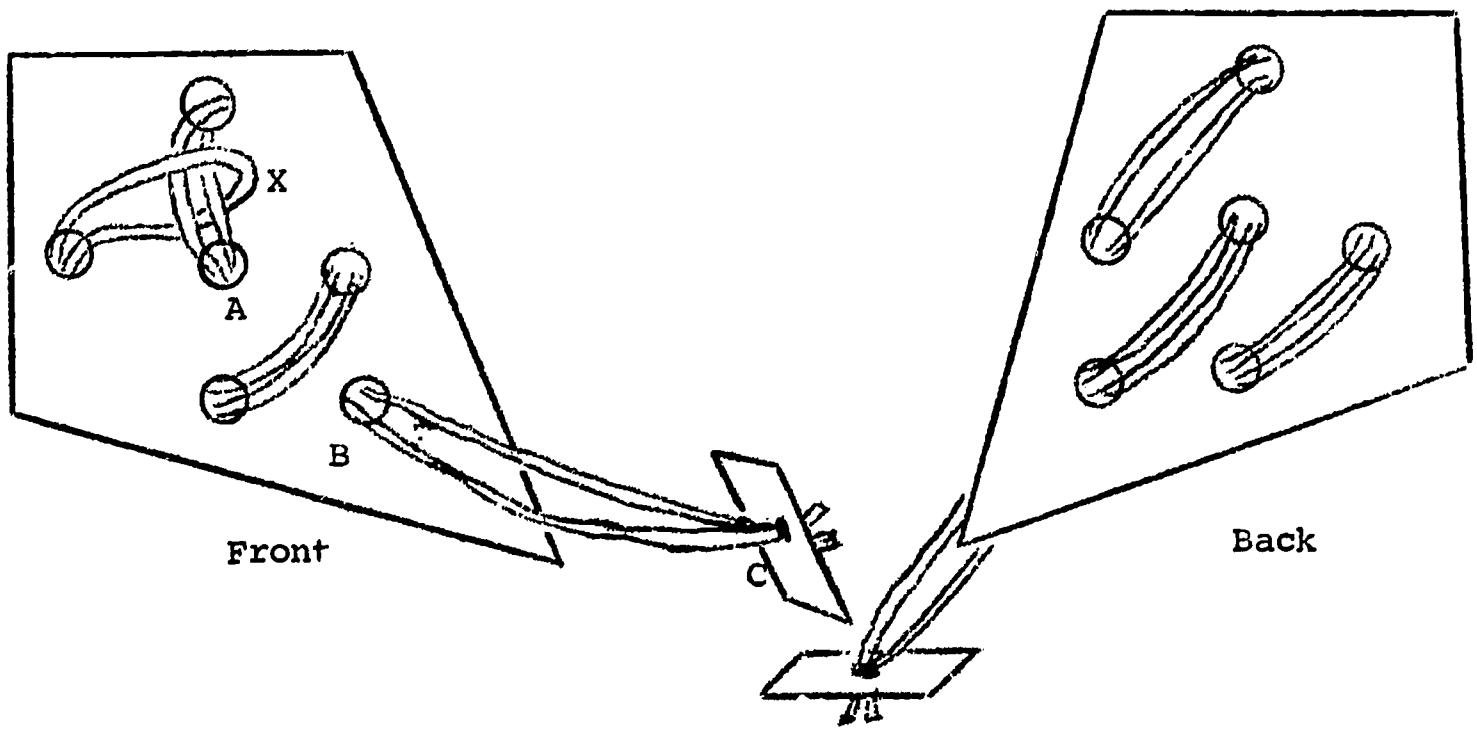
Can you do it?

Briefly describe your solution on a response sheet.



Solution: Pass the strip formed by the two cuts through the hole allowing the string to be put on or taken off without removing the buttons.

KITE KNOT



Material: A piece of string or a leather shoe lace about 24" long and a piece of wood ¼" × 4" × 5" or a piece of heavy cardboard, and a metal washer or a 1" × 1" × ¼" piece of wood.

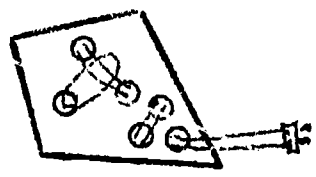
Preparation: Drill six holes in the piece of wood, which has been cut in the shape of a kite as in the pattern above. Thread the shoe lace through the holes as in the diagram and tie the ends together after passing them through the washer or wood retainer. This piece prevents the knot from being pulled through the holes in the kite.

ESP KITE KNOT T - 3

It is possible to remove the kite string from the kite without untieing the knot, passing the knot through the holes, or breaking the string.

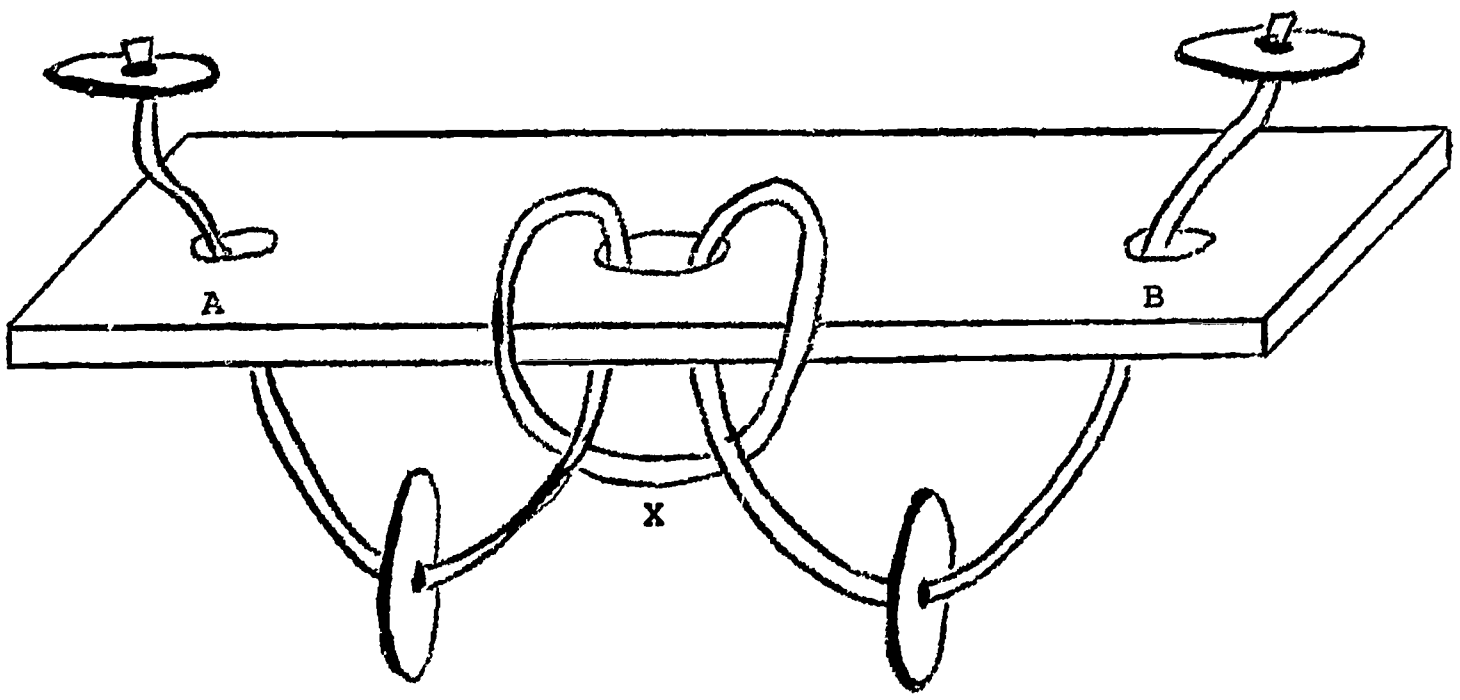
Can you do it?

Briefly describe your solution on a response sheet.



Solution: Thread the loop (X in the diagram) through holes A and B and around the knot, C. Pulling it back to its original position will allow the string to be removed completely.

THE GORDIAN KNOT



Materials: A piece of wood ¼" × 3" × 5", or leather, cardboard, floor tile, etc., a piece of string or leather shoe lace about 24" long, and four metal washers or beads that are larger than the holes.

Preparation: Drill three ½" holes in the piece of wood. Thread the shoe lace through the holes as in the pattern above, placing two of the washers at the ends of the string and the other two in the middle.

ESP THE GORDIAN KNOT T - 4

Alexander the Great, as a boy, was given a puzzle to test his cleverness. The puzzle was to untie the Gordian Knot. The legend was that whoever could untie the Gordian Knot would rule all Asia.

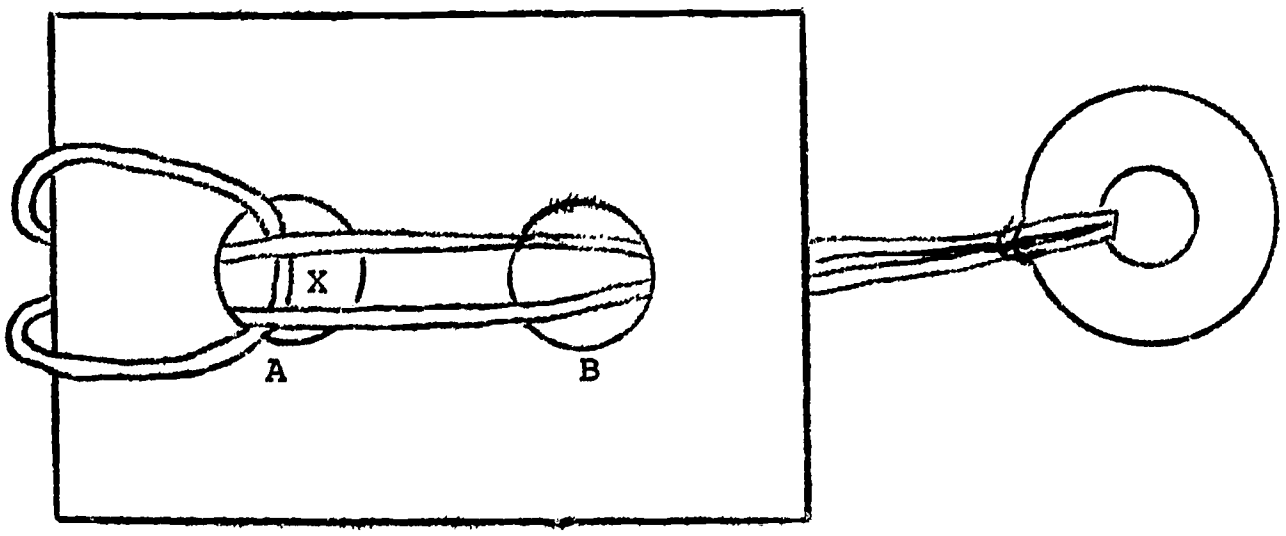
Alexander took one look at the puzzle, took out his sword and cut the knot.

Can you untie it without cutting it? When the knot is untied, the two washers will come together. You may not untie the ends or break the string.

Describe your solution on a response sheet.

Solution: Thread the loop (X in the diagram above) through hole A and around the washer. Repeat for hole B.

ANOTHER KNOT



Materials: A piece of wood ¼" × 2" × 3" or heavy cardboard, a piece of string or leather shoe lace about 12" long, and a metal washer or large button.

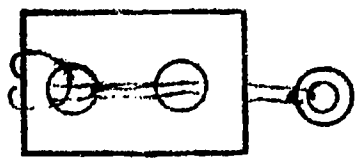
Preparation: Drill two ½" holes in the piece of wood and thread the string through the holes as in the pattern above, tieing the ends to the washer.

ESP ANOTHER KNOT T - 5

The object of this puzzle is to remove the string from the holes without untieing the ends of the string, breaking the string, or forcing the washer through the holes.

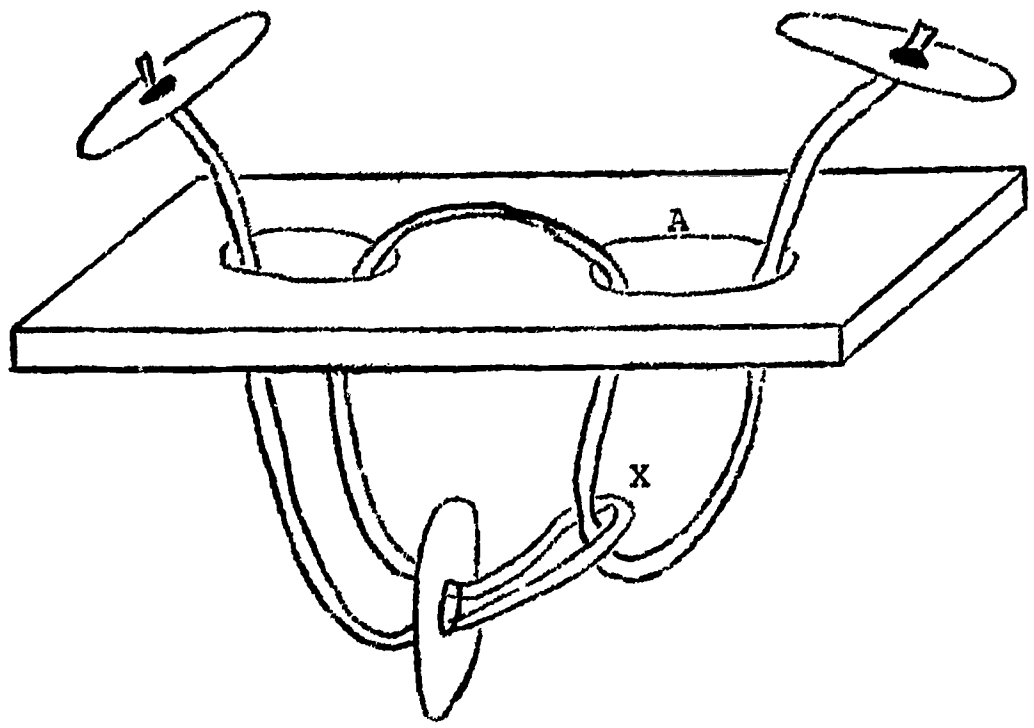
Can you do it?

Describe your solution on a response sheet.



Solution: Thread the loop (X in the diagram above) through hole B and pass the washer through it. Pulling the loop back to hole A will leave the string free and easy to remove.

RING RELEASE



Materials: A piece of wood ¼" × 3" × 4" or a piece of heavy cardboard, 3 metal washers (larger than the holes) and a piece of string or leather shoe lace about 24" long.

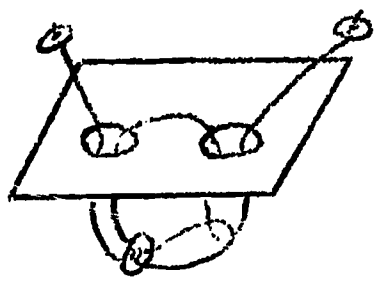
Preparation: Drill two ½" holes in the wood and thread the shoe string through as in the diagram above, tieing two of the washers on the ends.

ESP RING RELEASE T - 6

The object of the puzzle is to remove the ring (metal washer) from the string without untieing the ends, breaking the string, or forcing any of the washers through the holes.

Can you do it?

Describe your solution on a response sheet.



Solution: Thread loop X through hole A and pass the washer through the loop. Pulling it back through the hole will release the washer.

VI: MISCELLANEOUS PUZZLES

1) TOWER OF HANOI

2) PUZZLING PYRAMID

3) DOUBLE CROSS

4) THREE X THREE

5) FIVE X FIVE

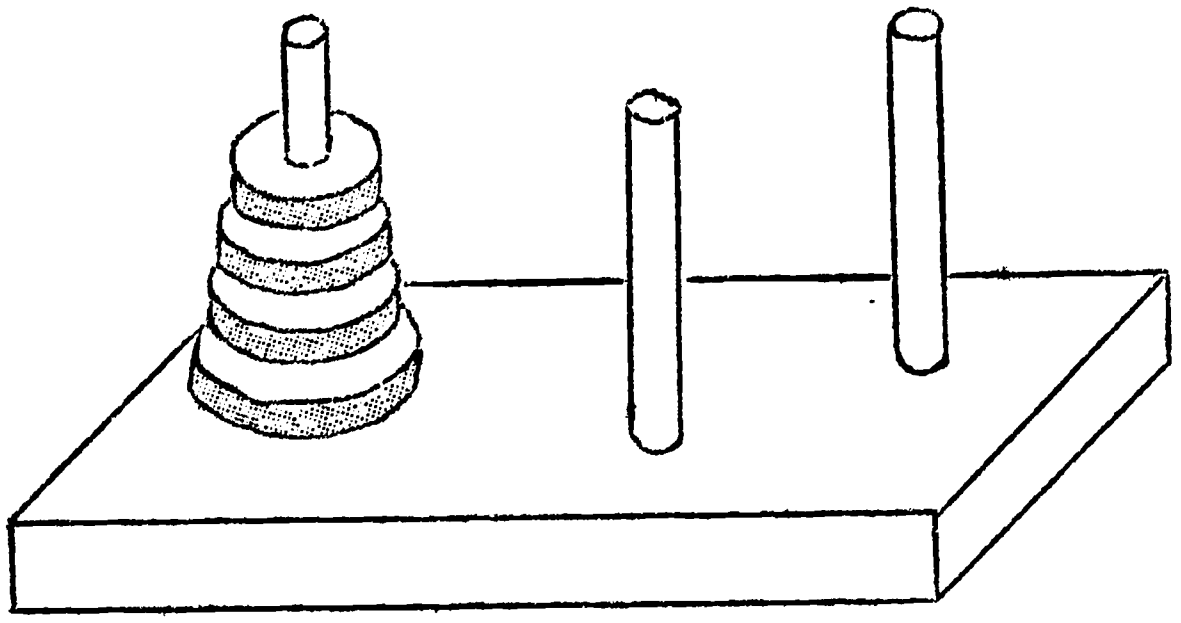
6) STICK PUZZLES

7) LOCO

8) DIGIT

9) MYSTERY MAZE

TOWER OF HANOI



Material: A block of wood 1" × 4" × 4", three ¼" dowel sticks 2" in length, and six metal washers of different sizes from large to small.

Preparation: Drill three ¼" holes in the block in a triangular pattern. Insert the dowels and stack up the washers on one of the pegs.

Note: If it is not possible to have a classroom set made, the same thing can be done using cardboard discs on a paper playing board.

ESP TOWER OF HANOI M - 1 A

Object: Move the pyramid of discs from one peg to another.

Rules: 1) You may only move one disc at a time.

2) You may never put a large disc on top of a smaller one.

Try to do it in as few moves as possible.

It is recommended that you start out with just four discs. As you perfect your system add more discs.

ESP TOWER OF HANOI M - 1 B

There is a formula that will tell you the minimum number of moves it takes to move all the discs from one peg to another. It is: $2^{\square} - 1 = \Delta$

If you fill in the box with the number of discs you are working with and then compute the expression it will give you the number of moves, in the triangle.

How many moves does it take with 3 discs?
How many moves does it take with 4 discs?
How many moves does it take with 5 discs?
How many moves does it take with 6 discs?

Class Procedure:

1) Introduce the puzzle by telling the story of its history:

The Tower of Hanoi was discovered hundreds of years ago at a Buddhist monastery near Hanoi, being used as a means of predicting the end of the world. Around the clock teams of Buddhists moved a stack of one hundred golden discs from one silver needle to another according to a prescribed ritual. They felt that when the stack had been transferred from one needle to another, the world would end.

2) Pass out a set of towers to the class and explain the object of the puzzle and the rules involved:

1. You may only move one disc at a time.
2. You may never put a large disc on top of a smaller one.

3) After the class has had time to experiment and, hopefully, discover a solution, put the table opposite on the board and have them fill in the correct numbers. The correct answers are in the parentheses.

# of discs	# of moves
0	(0)
1	(1)
2	(3)
3	(7)
4	(15)
5	(31)
6	(63)

Do you see a pattern?

How many moves would eight discs take?

Does the formula $2^{x} - 1 = y$, where x is the number of discs and y is the number of moves, fit the information in the table?

Can you use it to find new information?

How many moves would it take to move a hundred discs?

If they moved one disc a second, how long would it take the Buddhists to bring about the end of the world?

4) Some find it difficult to always move the right disc to the right tower thus getting the minimum number of moves, especially when the number of discs is increased. To eliminate guess work, a set of instructions has been devised by writing the number of each move in base two notation and then interpreting the position of the 1's and 0's.

Move the _____ (disc named by the first 1, reading from right to left)

disc to the

tower _____ (occuppied by -- if the number of zeros between the two 1's is <u>even</u>)

by the (not occuppied by -- if <u>odd</u>)

_____ (disc named by the second 1, reading from right to left)

disc.

To use this system, we must number the discs 1, 2, 3, ···, beginning with the smallest disc.

For example, the tenth move, written 1 0 1 0 tells us to move the <u>2nd</u> disc (since the first 1 appears in the second position) to the tower *not occuppied by* (the number of zeros between the two 1's is odd) the <u>4th</u> disc (since the second 1 appears in the fourth position).

Thus, the first few moves would be as follows:

1st	1	- move the 1st disc to an empty tower.
2nd	1 0	- move the 2nd disc to the remaining empty tower.
3rd	1 1	- move the 1st disc to the tower occupied by the 2nd disc (0 is even)
4th	1 0 0	- move the 3rd disc to an empty tower.
5th	1 0 1	- move the 1st disc to the tower not occupied by the 3rd disc.
6th	1 1 0	- move the 2nd disc to the tower occupied by the 3rd disc.
19th	1 0 0 1 1	- move the 1st disc to the tower occupied by the 2nd disc.
20th	1 0 1 0 0	- move the 3rd disc to the tower not occupied by the 5th disc.

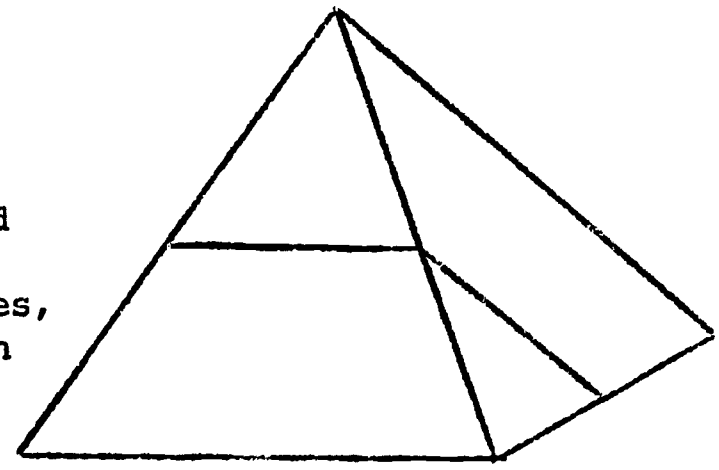
Solution: The complete solution is left up to you, but may be written out by continuing the listing above.

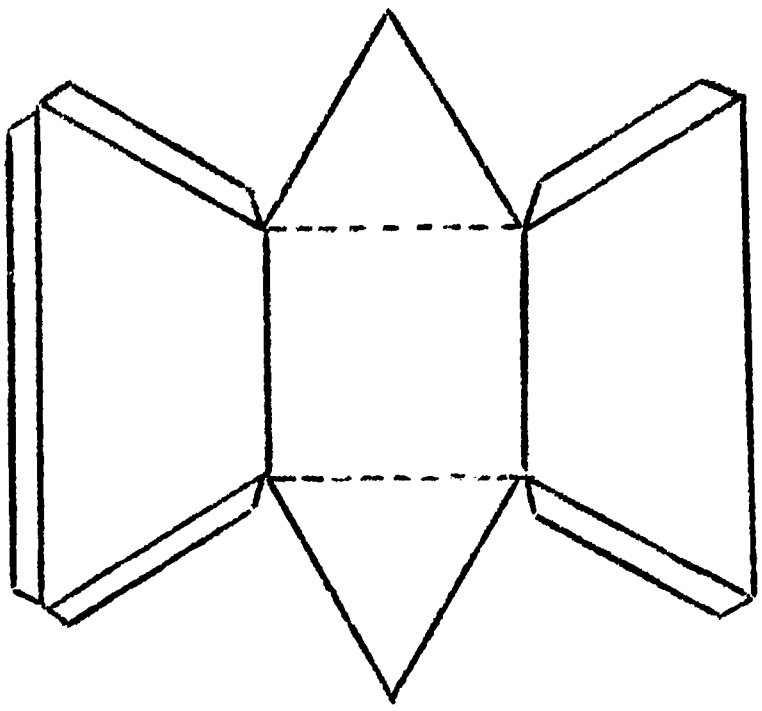
Reference: Kraitchik, <u>Mathematical Recreations</u>.

PUZZLING PYRAMID

Material: Oak tag or heavy cardboard and scotch tape.

Preparation: Enlarge the pattern below so that the base of the trapezoid is 6" and the sides of the square are 3". Make two pieces, cut out and tape together with the flaps on the inside.





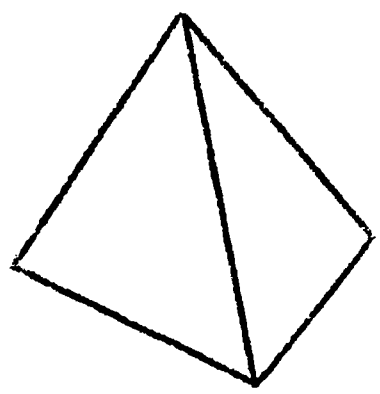
This puzzle can also be made using four identical pieces. However, it is the same puzzle with each piece cut in half.

ESP PUZZLING PYRAMID M - 2

It is possible to make a regular Egyptian pyramid (tetrahedran) having triangular shaped sides and base.

Can you do it?

Sketch the position of the pieces in your solution on a response sheet.



Source: S. S. Adams Co.
P. O. Box 369
Neptune, New Jersey 07753

Game: Pyramid Puzzle
Material: Plastic (4 pieces)
Price: $.35

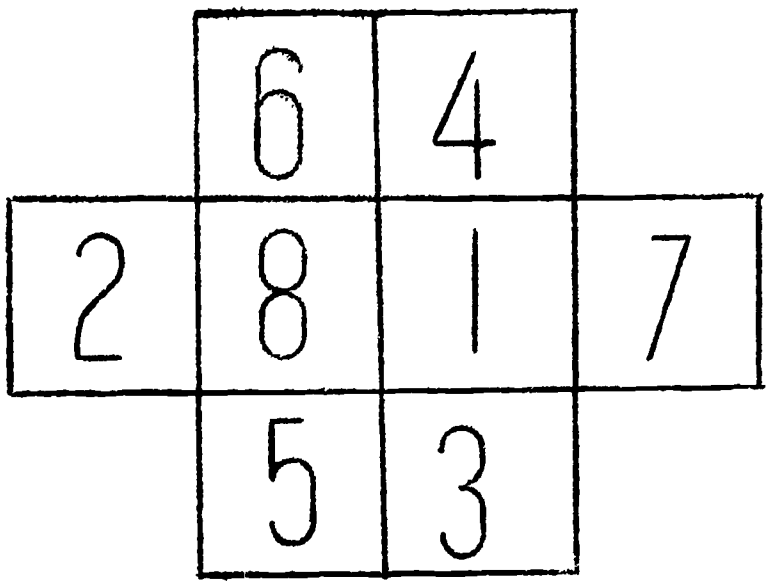
DOUBLE CROSS

Material: Oak tag or cardboard.

Preparation: Cut out eight squares 2" × 2" and number from one to eight. One number on each square.

If you write on both sides of each square, they will never have to be turned over.

If you prefer to use a playing board, draw the figure, leaving out the numbers, on another piece of cardboard or on a piece of wood.



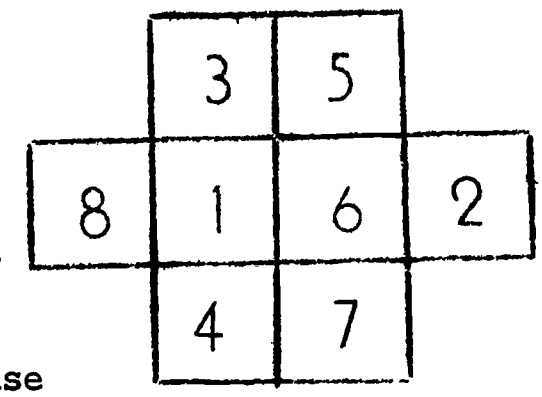
ESP DOUBLE CROSS M - 3

Arrange the squares as in the pattern below so that no consecutive integers (i.e. 3 and 4) are touching either on a side or on a corner.

The arrangement opposite is not correct because 5, 6, and 7 are touching on a side.

Can you come up with the correct arrangement?

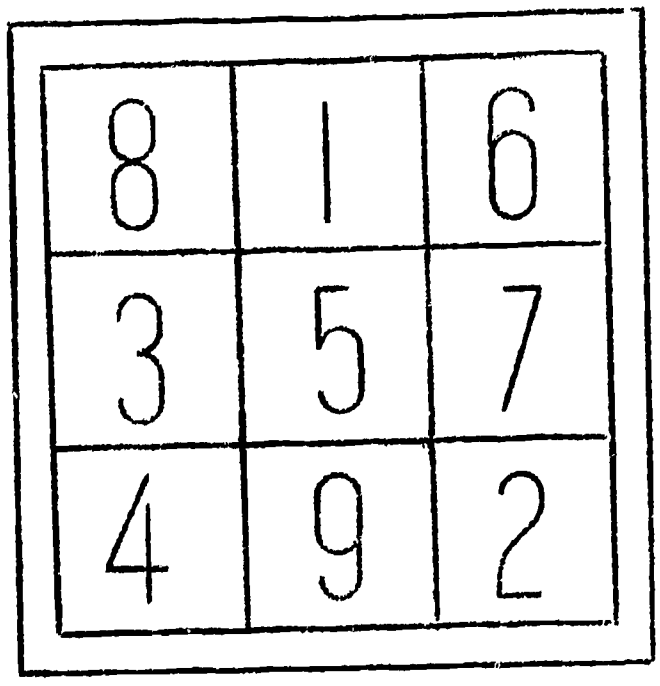
Sketch your solution on a response sheet.



THREE X THREE

Material: Oak tag or cardboard.

Preparation: Cut out nine squares 2" × 2" and number them from one to nine, one number per square. Using the same number on both sides eliminates having to turn any squares over.



A playing board can be made by drawing the pattern of squares opposite on a piece of cardboard or wood. This idea can be extended to larger magic squares by simply increasing the number of squares.

The game can also be made using pegboard and golf tees with the numbers painted or pasted on their tops.

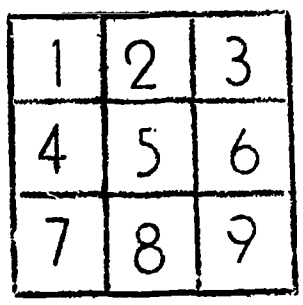
ESP THREE X THREE M - 4

Arrange the nine small squares to make a 3 × 3 square, such that the sum of each row, each column, and both diagonals is equal.

What is the correct sum for each row?

Is there a pattern in your solution that would help you to make a 5 × 5, or a 7 × 7 magic square?

Sketch your solution on a response sheet. Is there more than one solution to this problem?



Solution: The one given above is the only unique solution to this puzzle. All others are merely reflections or rotations or both.

Source: Are-Joy Game Co., Inc.
7509 Denison Avenue
Cleveland, Ohio 44102

Game: Perfect Squares
Material. Plastic and wood
Price: $1.00

FIVE X FIVE

Material: Oak tag or cardboard.

Preparation: Cut out twenty-five squares 2" × 2" and number them so as to end up with five 1's, five 2's, etc. If you put the same number on both sides of a square, it will never have to be turned over.

A playing board can be made out of another piece of cardboard or a piece of wood marked off with squares as it is in the pattern opposite.

This game can also be made using pegboard and golf tees with numbers pasted or painted on their tops.

4	5	1	2	3
1	2	3	4	5
3	4	5	1	2
5	1	2	3	4
2	3	4	5	1

ESP

FIVE X FIVE

M - 5

Arrange the twenty-five small squares into a 5 x 5 square, so that each row, each column, and both diagonals add up to the same number. This is a special type of magic square.

What is the sum of each row?

Sketch your solution on a response sheet.

Is there a pattern to your solution?

1	2	3	4	5
1	2	3	4	5
1	2	3	4	5
1	2	3	4	5
1	2	3	4	5

STICK PUZZLES

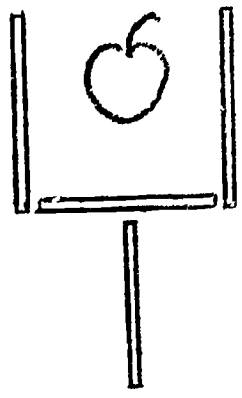
Material: Two dozen toothpicks or match sticks which have had the ends cut off.

Preparation: Enclose the sticks and instruction cards in an envelope or small box and the puzzle is complete.

ESP STICK PUZZLES M - 6 A

Arrange four sticks and a marker (a coin or paper clip) as in the figure below. By moving just two sticks, turn the cocktail class upside down so that the cherry is on the outside.

Sketch your solution on a response sheet.



ESP STICK PUZZLES M - 6 B

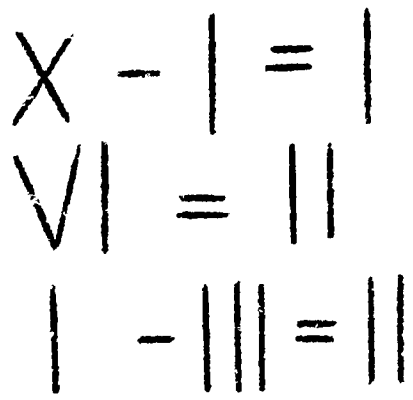
The thirteen sticks in the figure below represent fences that make six cattle pens. If one fence (stick) were removed, could you still make six pens? or separate compartments?

Sketch your solution on a response sheet.

ESP STICK PUZZLES M - 6 C

By moving just one stick in each equation, can you make a true statement? You may use Roman Numerals.

Record your solution on a response sheet.

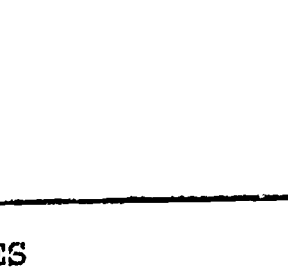


ESP STICK PUZZLES M - 6 D

Arrange twelve sticks as in the figure below.

1) By removing just two sticks, leave just two squares.

   Show your solution on a response sheet.



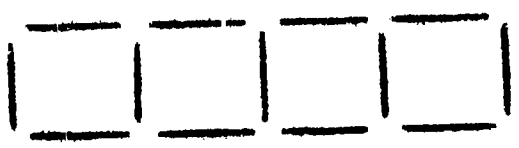
2) Using the same figure, can you make three squares by moving four of the sticks?

ESP STICK PUZZLES M - 6 E

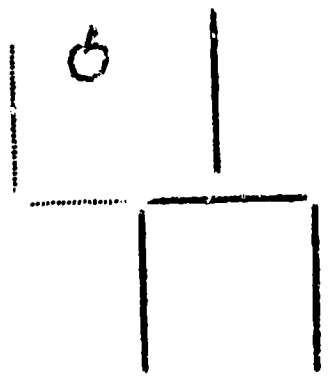
Arrange thirteen sticks in the pattern shown below.

Remove one stick and move three others so as to spell out the answer to the question: What are matches made of?

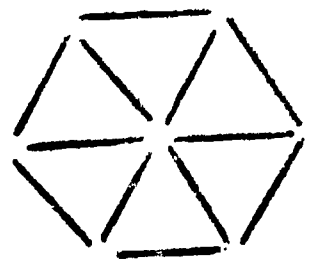
Sketch your answer on a response sheet.



Solutions: A)



B)

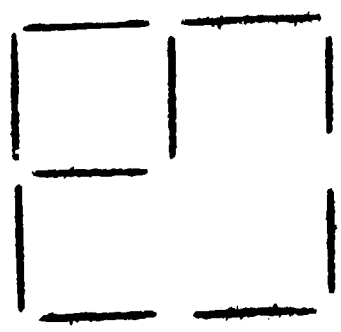


C) 1) || − | = |

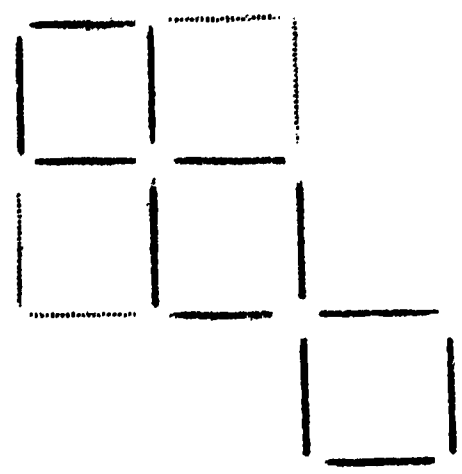
2) X| = ||

3) | = ||| − ||

D) 1)



2)

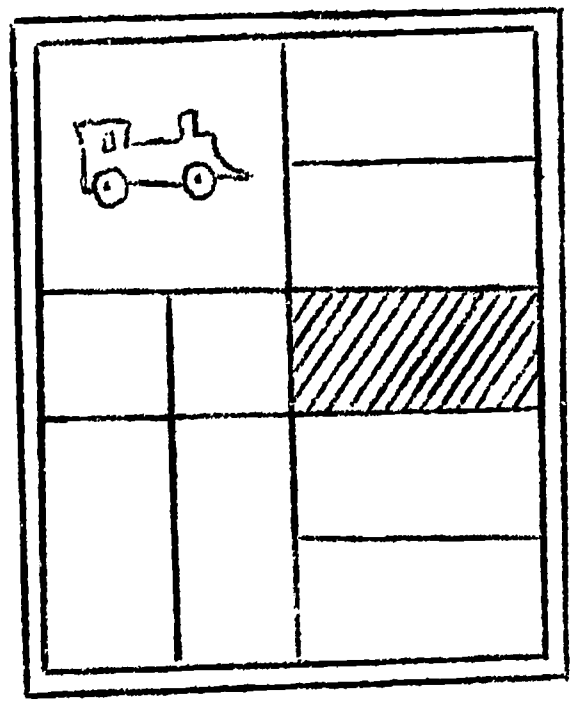


E)

For other puzzles which could be added to these, check the references below. This particular set of puzzles works well with an entire class. Solutions are best demonstrated on an overhead projector.

References: Mathematical Puzzles and Pastimes, Aaron Bakst.
Fun With Puzzles, Joseph Leeming.

Material: Plastic puzzle.



ESP LOCO M - 7

Object: To move the large square from the upper left hand corner to the lower right hand corner without lifting any of the other pieces.

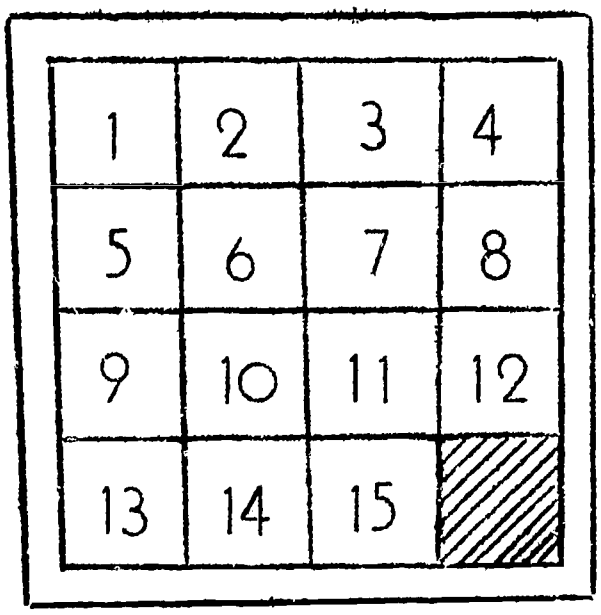
Source: E. S. Lowe Company Inc.
200 - 5th Avenue
New York, New York 10010

Material: Plastic
Cost: $.49

DIGIT

Material: Plastic puzzle.

(also called:
Boss Puzzle and 15 Puzzle)



ESP DIGIT M - 8

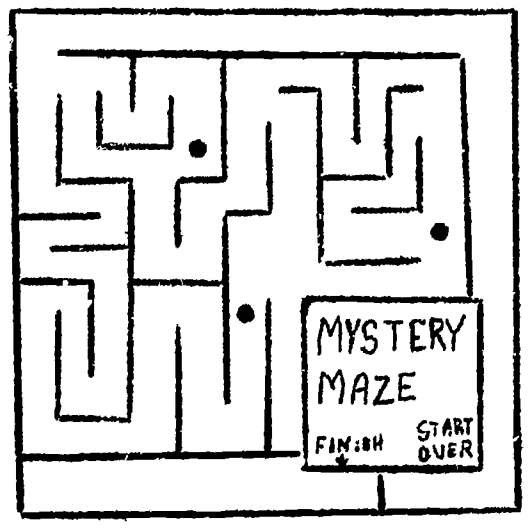
Object: To arrange the fifteen digits in various patterns. There are 6! combinations only half of which are possible.

Source: Roalex Company
Forest Hills, New York
11375

Material: Plastic
Cost: $.49

MYSTERY MAZE

Material: Plastic puzzle.



ESP	MYSTERY MAZE	M - 9

Object: To find the quickest way of going from start to finish.

Source: Harmonic Reed Corporation
Union Hill Road
West Conshohocken
Pennsylvania 19428

Material: Plastic
Cost: $.39